Worksheet 7C

Random variable: a *quantity* that can take different values in different outcomes.

Expected value: the long-run average value of a random variable.

$$E(X) = v_1 \cdot P(X = v_1) + v_2 \cdot P(X = v_2) + \ldots + v_n \cdot P(X = v_n)$$

where $v_1, v_2, \ldots, v_n$ are all possible values of the random variable X.

Law of Large Numbers: Consider an event A with probability $P(A)$ in a single trial. For a large number of trials, the proportion of trials in which event A occurs is approximately the probability $P(A)$. The larger the number of trials, the closer the proportion is to $P(A)$.

Gambler's fallacy: (also called *gambler's ruin*) the mistaken belief that a streak of bad luck makes a person due for a streak of good luck (or that a streak of good luck will continue).

1) You draw a card from a deck of 52 card. If it is a face card (Jack, Queen, King), you win 2 dollars. Otherwise, you lose 50 cents. In the long run, do you win or lose? How much do you win or lose per game on average?

2) You roll a dice and calculate your score as follows. Your score is the number you roll if it is 4, 5, or 6. If you roll 1, 2, or 3, you get to roll one more time and your score is the sum of the two numbers. What is the expected value of your score?

3) An insurance policy sells for \$325. Based on past data, an average of 1 in 100 policyholders will file a \$10,000 claim, an average of 1 in 250 policyholders will file a \$25,000 claim, and an average of 1 in 500

policyholders will file a $50,000 claim. Find the expected value (to the company) per policy sold. Find the expected profit if the company sells 1000 polices and if it sells 100,000 policies.

4) Suppose you play a coin toss game in which you win $1 if a head appears and lose $1 if a tail appears. In the first 100 coin tosses, heads comes up 46 times and tails comes up 54 times. What percentage of times has heads come up in the first 100 tosses? What is your net gain or loss at this point?

Suppose you toss the coin 200 more times (a total of 300 tosses), and at that point heads has come up 47% of the time. Is this increase in the percentage of heads consistent with the law of large numbers? What is your net gain or loss at this point?

How many heads would you need in the next 100 tosses in order to break even after 400 tosses? Is it reasonable to expect this to occur?

Suppose that, still behind after 400 tosses, you decide to keep playing because you are “due” for a winning streak. Explain how this belief would illustrate the gambler’s fallacy.