Final Exam: Some problems for review

The exam will be taken in class (SCB 303) on Wednesday 4/15 from 2 – 4 PM. You will do the exam on paper. You can use a pocket calculator of any kind. Phones, laptops, and notecards are not allowed. The instructor will provide scratched papers for you.

Other exam policies:

The proctor may reassign your seat at the beginning or at any time during the exam.

Using a phone or any unauthorized assistance while the exam is in progress, whether inside or outside of the classroom, is prohibited.

If you need to leave the room for any reason, you must first obtain the proctor's permission. If the proctor is not present in the room and you want to leave, you must wait until he/she comes back.

Violation of any of the above policies is considered as cheating and may result in a score of zero.

The topics to be covered are

- Solve a system of linear equations exactly (using Gauss elimination method) and approximately (using Gauss-Jacobi method),
- Solve a differential equation with boundary conditions using finite difference method.

The following formulae will be provided on the exam:

$$y'(x_k) = \frac{y_{k+1}-y_k}{h} + O(h) = \frac{y_k-y_{k-1}}{h} + O(h) = \frac{y_{k+1}-y_{k-1}}{2h} + O(h)$$

$$y'(x_k) = \frac{-3y_k+4y_{k+1}-y_{k+2}}{2h} + O(h^2) = \frac{y_{k-2}-4y_{k-1}+3y_k}{2h} + O(h^2) = \frac{y_{k+1}-y_{k-1}}{2h} + O(h^2)$$

$$y''(x_k) = \frac{y_k-2y_{k+1}+y_{k+2}}{h^2} + O(h) = \frac{y_{k-2}-2y_{k-1}+y_k}{h^2} + O(h) = \frac{y_{k+1}-2y_k+y_{k-1}}{h^2} + O(h)$$

$$y''(x_k) = \frac{2y_k-5y_{k+1}+4y_{k+2}-y_{k+3}}{h^2} + O(h^2) = \frac{-y_{k-3}+4y_{k-2}-5y_{k-1}+2y_k}{h^2} + O(h^2) = \frac{y_{k+1}-2y_k+y_{k-1}}{h^2} + O(h^2)$$

You should review the homework problems, worksheets, quizzes, examples given in the lectures. It is always a good idea to study for the exam with someone.

Some problems to practice:

1) Solve the system

$$\begin{cases} 3x_1 + x_2 + x_3 = 1 \\ -x_1 + 4x_2 - x_3 = 2 \\ x_1 - x_2 + 3x_3 = 3 \end{cases}$$

(a) Using Gauss elimination method

(b) Using Gauss Jacobi method

2) Consider the boundary value problem $xy'' - y' + y = x^2$, $y(0) = 2$, $y(1) = 1$.

(a) With nodal points $0 = x_0 < x_1 < \cdots < x_n = 1$ and $y_k = y(x_k)$, set up the difference equations at order $O(h^2)$ for the problem.

(b) Solve the boundary value problem using $h = 0.25$.

3) Let $R = [1, 2] \times [0, 3]$. Consider the Laplace equation $u_{xx} + u_{yy} = 0$, with boundary values $u(1, y) = y$, $u(2, y) = 0$, $u(x, 0) = 0$, $u(x, 3) = 6 - 3x$.

(a) With nodal points $1 = x_0 < x_1 < \cdots < x_n = 2$, $0 = y_0 < y_1 < \cdots < y_m = 3$ and $u_{jk} = u(x_j, y_k)$, set up the difference equations at order $O(h^2)$ for the problem.

(b) Solve the problem using $m = n = 3$.