Lecture 12: Interpolation with evenly spaced points (02/06/2026)

0.8 Newton's Forward Divided Difference

Newton's Forward Divided Difference method is a way to find a **polynomial formula** that matches a set of given data points when the x-values increase by the same amount each time. It uses **forward differences**, which are the changes in the y-values as x increases, to see how the values are changing. Then it uses this pattern to build the polynomial step by step, adding new terms so the final formula fits all the given data points exactly.

Importance: It helps us approximate derivative of a function.

Recall: $(x_0, y_o), (x_1, y_1), \ldots, (x_n, y_n)$

$$P(x) = c_o + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \ldots + c_n(x - x_0) \ldots (x - x_{n+1})$$

$$c_k = f[x_0, x_1, \ldots, x_k] \quad \text{k'th Newton divided difference}$$

$$c_o = f[x_0] = f(x_0)$$

Special Case: $x_0, x_1, \ldots, x_n$ are equally spaced

$$x_0 = x_0$$

$$x_1 = x_0 + h$$

$$x_2 = x_0 + 2h$$

$$\vdots$$

$$x_n = x_0 + nh$$

$$f(x) \approx P(x) = c_o + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \ldots + c_n(x - x_0) \ldots (x - x_{n+1})$$

If h is small enough and x is close to x_0 then

$$f(x) \approx c_o + c_1(x - x_0) + c_2(x - x_0)^2 + \ldots + c_n(x - x_0)^n$$

<u>Recall</u>: Taylor Polynomial

$$f(x) \approx T_n(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

So,

$$c_k \approx \frac{f^{(k)}(x_0)}{k!}$$

$$\boxed{f^{(k)}(x_0) \approx k!\ c_k}$$

0.9 Backward Divided Difference

Newton's Backward Divided Difference method is a way to find a polynomial formula that fits a set of given data points when the x-values increase by the same amount each time. Instead of starting from the first data point like the forward method, it starts from the last data point and works backward using backward differences. These differences show how the y-values change as we move backward through the data. The polynomial is then built step by step using this pattern so that the final formula passes through all the given data points exactly.

Importance: It is useful when we want to approximate values or derivatives of a function near the end of the data set.

$x_0 < x_1 < \ldots < x_n$

We want to approximate $f'(x_n), f''(x_n) \ldots$

Reorder the sequence: $x_n, x_{n-1}, x_{n-2}, \ldots, x_0$

Then use forward divided difference for this.