Lecture 13: Spline Interpolation and Cubic Splines (02/13/2026)

Range phenomenon: high oscillation near the endpoints in polynomial interpolation.

$$\epsilon = \max_{[a,b]} |f(x) - P(x)| \leq \Big(\max_{[a,b]} |f^{(n+1)}(x)| \Big) |x - x_0||x - x_1| \dots |x - x_n| \Big)$$

Spline Interpolation

We interpolate the data points by a piecewise polynomial function. This function is called a spline function if the following conditions are met.

1. The curve that joins x_i and x_{i+1} is a polynomial of deg $\leq$ k.
2. The piecewise function is continuous and the first (k-1)'th derivatives are also continuous.