Lecture 14: Spline Functions Continuation (02/18/2026)

Example 1: Checking for quadratic spline

$$f(x) = \begin{cases} -x, & -1 \leq x \leq 0 \\ x^2, & 0 < x \leq 1 \end{cases}$$

$$f'(x) = \begin{cases} -1, & -1 \leq x \leq 0 \\ 2x, & 0 < x \leq 1 \end{cases}$$

$$= \begin{cases} f(0^-) = 0 \\ f(0^+) = 0 \end{cases}$$

$$\lim_{x \to 0^-} f'(x) = -1, \quad \lim_{x \to 0^+} f'(x) = 0$$

So f' has a jump at $x = 0$ and the function is not continuous. So, f is not a quadratic spline.

Example 2: Linear Spline Function

Find the linear spline function using the given points $(0, 1), (1, 3), (2, 2)$

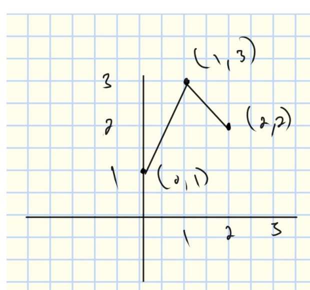

Figure 5: Linear Spline

$$m_1 = \frac{3-1}{1-0} = 2 \quad m_2 = \frac{2-3}{2-1} = -1$$

$$y - 1 = 2(x - 0) \rightarrow y = 2x + 1$$

$$y - 3 = -1(x - 1) \rightarrow y = -x + 4$$

$$\boxed{f(x) = \begin{cases} 1 + 2x, & 0 \le x \le 1 \\ -x + 4, & 1 \le x \le 2 \end{cases}}$$

Example 3: Quadratic Spline Function

Find the quadratic spline function using the same points in (**Example 2**).

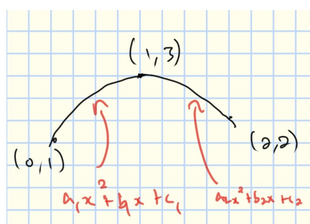

Figure 6: Quadratic Spline

$$S_1(x) = a_1(x)^2 + b_1(x) + c_1, \quad 0 \le x \le 1$$

$$S_2(x) = a_2(x)^2 + b_2(x) + c_2, \quad 1 \le x \le 2$$

$$a_1(0)^2 + b(0) + c_1 = 1 \rightarrow c_1 = 1$$

$$a_1(1)^2 + b(1) + c(1) = 3 \rightarrow a_1 + b_1 + 1 = 3 \rightarrow a_1 + b_1 = 2$$

$$a_2(1)^2 + b_2(1) + c_2 = 3 \rightarrow a_2 + b_2 + c_2 = 3$$

$$a_2(2)^2 + b_2(2) + c_2 = 2 \rightarrow 4a_2 + 2b_2 + c_2 = 2$$

$$2a_1 + b_1 = 2a_2 + b_2$$

$$b_1 = 0$$

$$a_1 + 0 = 2 \rightarrow a_1 = 2$$

$$2a_1 + b_1 = 4 \rightarrow 2a_2 + b_2 = 4 \rightarrow b_2 = 4 - 2a_2$$

$$(4a_2 + 2b_2 + c_2) - (a_2 + b_2 + c_2) = 2 - 3 \rightarrow 3a_2 + b_2 = -1$$

$$3a_2 + (4 - 2a_2) = -1 \rightarrow a_2 = -5$$

$$b_2 = 4 - 2(-5) = 14$$

$$a_2 + b_2 + c_2 = 3 \rightarrow -5 + 14 + c_2 = 3 \rightarrow c_2 = -6$$

$$f(x) = \begin{cases} 2x^2 + 1, & 0 \le x \le 1 \\ -5x^2 + 14x - 6, & 1 \le x \le 2 \end{cases}$$