

## Lecture 15: Numerical Differentiation (02/20/2026)

Suppose that a function  $f$  passes through  $(x_0, y_0), \dots, (x_n, y_n)$

$$f(x) \approx P(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \dots + c_n(x - x_0) \dots (x - x_{n-1})$$

$$c_k = f[x_0, x_1, \dots, x_k]$$

$$f'(x_0) \approx P'(x_0) = c_1 + c_2((x - x_1) + (x - x_0)) + c_3((x - x_0)(x - x_1)(x - x_2) + (x - x_0)(x - x_1)(x - x_3) + \dots)$$

$$f''(x_0) \approx P''(x_0) = 2c_2 + 2c_3((x - x_0) + (x - x_1) + (x - x_2)) + 2c_4((x - x_0)(x - x_1) + (x - x_0)(x - x_2) + \dots)$$

### 0.10 Product Rule of Differentiation

$$(fg)' = f'g + fg'$$

$$(fgh)' = (fg)'h + fgh' \Rightarrow f'gh + fg'h + fgh'$$

$$(fghk)' = f'ghk + fg'hk + fgh'k + fghk'$$