Lecture 16: Approximation using Newton's forward divided difference (02/23/2026)

$$f(x_0) = y_o$$
$$\vdots$$
$$f(x_n) = y_n$$
$$f(x) \approx P(x) = c_o + c_1(x - x_0) + \ldots + c_n(x - x_0)\ldots(x - x_{n-1})$$
$$f'(x) \approx P'(x) = c_1 + c_2(x_0 + x_1) + c_3(x_0 - x_1)(x_0 - x_2) + \ldots$$
$$f''(x) \approx P''(x) = 2c_2 + c_3(2(x_0 - x_1) + 2(x_0 - x_2)) + c_4(2(x_0 - x_1)(x_0 - x_2) + 2(x_0 - x_1)(x_0 - x_3) + 2(x_0 -$$

<u>Special Cases</u>

$$f'(x_0) \approx P'(x_0) = c_1 + c_2(-h) + c_3(-h)(-2h) + c_4(-h)(-2h)(-3h) + \ldots$$
$$c_1 - hc_2 + 2!h^2c_3 - 3!h^3c_4 + O(h^4)$$
$$f''(x_0) \approx P''(x_0) = 2c_2 + (-6h)c_3 + 22c_4h^2 + O(h^3)$$

<u>Note</u>

$O(h^4)$ is $\alpha h^4 + \ldots h^5 + \ldots$

$= h^4 \underbrace{(\alpha + \ldots h + h^2 + \ldots)}_{\approx\alpha}$

$\approx \alpha h^4$

$f(h) = O(h^\alpha)$ as $h \to 0$

if $\left|\frac{f(h)}{h^\alpha}\right| \leq C$ (constant for any small h)

$f(h) = o(h^\alpha)$ as $h \to 0$

if $\lim\limits_{h\to 0} \frac{f(h)}{h^\alpha} = 0$