Lecture 17: Newton's Central Divided Difference (02/25/2026)

Central divided difference is a numerical method used to approximate derivatives using function values on both sides of a point. Instead of only using forward or backward points, it uses symmetric points like $x_0 - h$ and $x_0 + h$, which improves accuracy. Because the points are evenly spaced around the center, many error terms cancel out, making the approximation more reliable. This method is commonly used to estimate the first and second derivatives of a function when only discrete data values are available. The central divided difference formula typically has an error of order $O(h^2)$, which is more accurate than forward or backward difference methods.

0.10.1 Grid of Equally Spaced Points

Assume the function values are known at equally spaced points:

$x_{i+1} - x_i = h$

Example grid:

$\ldots, x_{k-2}, x_{k-1}, x_k, x_{k+1}, x_{k+2}, \ldots$

For central difference, the point where we approximate the derivative is the center point x_k.

Points used:

$x_k - h = x_{k-1}, \quad x_k + h = x_{k+1}$

0.10.2 Central Difference Approximation for First Derivative

The first derivative at x_k is approximated using symmetric points:

$$f(x_k) \approx \frac{f(x_k + h) - f(x_k - h)}{2h}$$

or equivalently

$$f(x_k) \approx \frac{f(x_{k+1}) - f(x_{k-1})}{2h}$$

Error Order:

$$f(x_k) \approx \frac{f(x_{k+1}) - f(x_{k-1})}{2h} + O(h^2)$$

This means the truncation error decreases proportionally to h^2.

0.10.3 Central Difference Approximation for Second Derivative

The second derivative is approximated by

$$f(x_k) \approx \frac{f(x_k + h) - 2f(x_k) + f(x_k - h)}{h^2}$$

or

$$f(x_k) \approx \frac{f(x_{k+1}) - 2f(x_k) + f(x_{k-1})}{h^2}$$

Error Order:

$$f(x_k) \approx \frac{f(x_{k+1}) - 2f(x_k) + f(x_{k-1})}{h^2} + O(h^2)$$

0.10.4 Why Central Difference is More Accurate

Central difference uses values on both sides of the evaluation point.

Because the formula is symmetric:

- odd-order error terms cancel
- accuracy increases

Error Comparison:

Method	Formula	Error
Forward Difference	$\frac{f(x_0+h)-f(x_0)}{h}$	$O(h)$
Backward Difference	$\frac{f(x_0)-f(x_0-h)}{h}$	$O(h)$
Central Difference	$\frac{f(x_0+h)-f(x_0-h)}{2h}$	$O(h^2)$

Central difference is therefore more accurate for the same step size.