

Lecture 18: Numerical Integration and the Trapezoid Rule (02/27/2026)

We want to approximate the definite integral

$$\int_a^b f(x) dx$$

Sometimes:

$f(x)$ is complicated and we only know values of $f(x)$ at discrete points

In these cases we use a **numerical integration method**.

Two common approaches:

- Riemann sums
- Approximating $f(x)$ by a polynomial

Example

$$f(x) \approx p(x)$$

and

$$\int_a^b f(x) dx \approx \int_a^b p(x) dx$$

Instead of integrating the complicated function $f(x)$ directly, we replace it with a simpler function whose integral is easy to compute.

0.11 Partition of the Interval

Divide $[a, b]$ into n sub-intervals.

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b$$

Step size is $h = x_{k+1} - x_k = \frac{b-a}{n}$

0.12 Trapezoid Rule

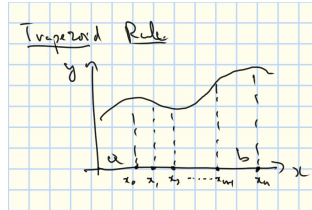
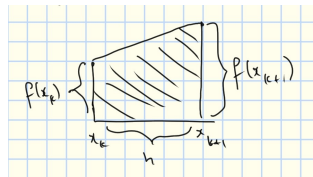


Figure 7: Trapezoid Rule

Approximate the curve on each interval with a straight line.

Area of each piece becomes a trapezoid.

$$\int_{x_k}^{x_{k+1}} f(x) dx \approx \frac{h}{2} [f(x_k) + f(x_{k+1})]$$



0.13 Composite Trapezoid Rule

Summing all trapezoids gives

$$\int_a^b f(x) dx \approx \frac{h}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Composite Trapezoid Rule

$$T_n = \frac{h}{2} \left[f(x_0) + 2 \sum_{k=1}^{n-1} f(x_k) + f(x_n) \right]$$

Example

Approximate $\int_1^2 \frac{1}{x} dx$ using several sub-intervals. Divide the interval and apply the trapezoid rule to each piece.