Lecture 19: Error Estimate of the Trapezoid Rule (03/02/2026)

Let $I = \int_a^b f(x)\,dx$ and let T_n be the trapezoid approximation.

We define the error $E_n = |I - T_n|$

The trapezoid rule approximates the function using a linear polynomial.Error depends on the curvature of the function, measured by the second derivative.

Let

$M = \max_{[a,b]} |f''(x)|$

For one interval:$\left|\int_c^d f(x)\,dx - \int_c^d p_1(x)\,dx\right| \leq \frac{M}{12}(d-c)^3$

For n sub-intervals with$h = \frac{b-a}{n}$ the total error satisfies $E_n \leq \frac{M}{12}(b-a)h^2$

Error bound for trapezoid rule

$$|E_r| \leq \frac{M(b-a)}{12n^2}$$

where $M = \max |f''(x)|$

Since the error behaves like $O(h^2)$ the trapezoid rule is a **second order method**