Lecture 2

Monday, January 12, 2026 1:47 AM

For the *bisection method*, when do we stop the iteration? It depends on the precision we want. Suppose the initial search interval is $[a, b]$. After halving this interval n times, the search interval is of length $\frac{b-a}{2^n}$. By taking the midpoint x_* of this interval as the approximate solution, we know that the error is at most $\frac{b-a}{2^{n+1}}$. That is,

$$|x_* - x_{ex}| \le \frac{b-a}{2^{n+1}}$$

Therefore, if we want the error to be less than some ϵ, we need to make sure n is large enough such that $\frac{b-a}{2^{n+1}} < \epsilon$.

Another way to terminate the iteration is when we see two consecutive iterations give the same result with a given number of decimal places. For example, suppose we want the result to be correct to 2 decimal places. We stop at x_n (the result at the n'th iteration) whenever x_{n-1} and x_n have the same 2 digits after the decimal point.

Implement on Matlab: (Live script)

bisection.mlx ×

/MATLAB Drive/bisection.mlx

Experiment with Bisection Method. We are solving the cubic equation

$$x^3 - 3x + 1 = 0$$

We want the solution between 0 and 1.

```
a = 0; b= 1;
for k = 1:5
    if f(a)*f((a+b)/2) < 0
        b = (a+b)/2;
    else
        a = (a+b)/2;
    end
end
(a+b)/2
```

```
ans = 0.3594
```

Define the function f

```
function y = f(x)
    y = x^3 - 3*x + 1;
end
```