

## Lecture 2

Monday, January 12, 2026 1:47 AM

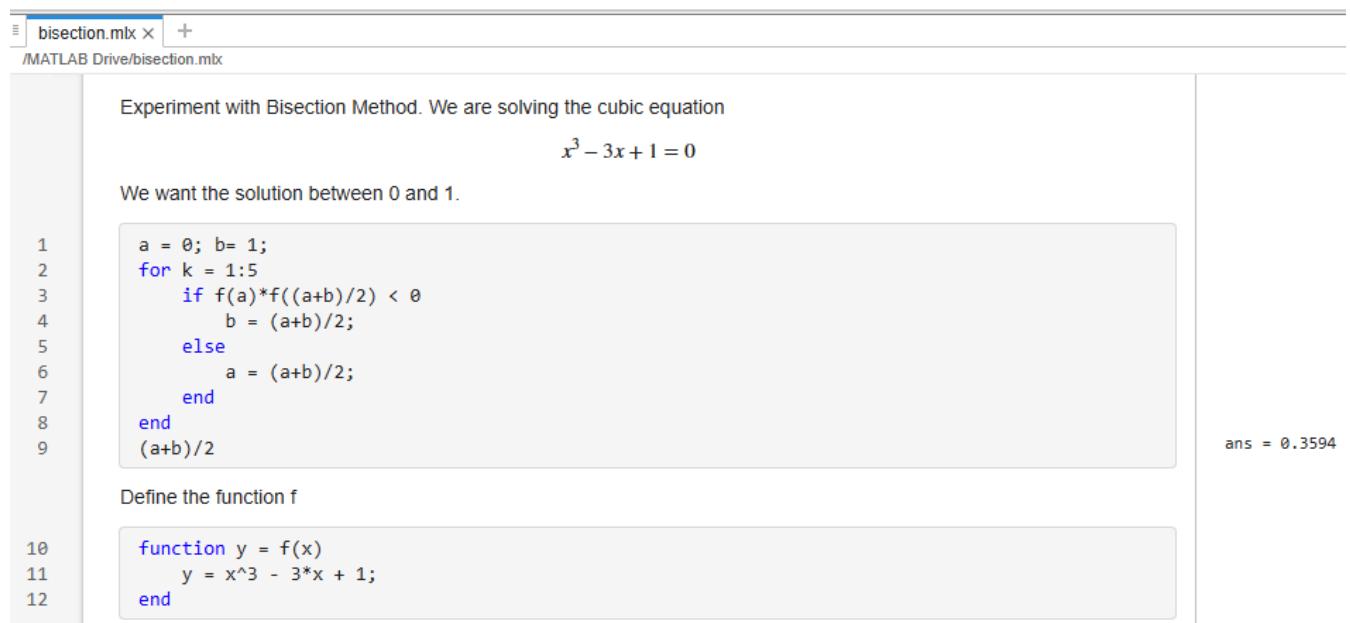
For the *bisection method*, when do we stop the iteration? It depends on the precision we want. Suppose the initial search interval is  $[a, b]$ . After halving this interval  $n$  times, the search interval is of length  $\frac{b-a}{2^n}$ . By taking the midpoint  $x_*$  of this interval as the approximate solution, we know that the error is at most  $\frac{b-a}{2^{n+1}}$ . That is,

$$|x_* - x_{ex}| \leq \frac{b-a}{2^{n+1}}$$

Therefore, if we want the error to be less than some  $\epsilon$ , we need to make sure  $n$  is large enough such that  $\frac{b-a}{2^{n+1}} < \epsilon$ .

Another way to terminate the iteration is when we see two consecutive iterations give the same result with a given number of decimal places. For example, suppose we want the result to be correct to 2 decimal places. We stop at  $x_n$  (the result at the  $n$ 'th iteration) whenever  $x_{n-1}$  and  $x_n$  have the same 2 digits after the decimal point.

Implement on Matlab: (Live script)



bisection mlx +

/MATLAB Drive/bisection mlx

Experiment with Bisection Method. We are solving the cubic equation

$$x^3 - 3x + 1 = 0$$

We want the solution between 0 and 1.

```
1 a = 0; b= 1;
2 for k = 1:5
3     if f(a)*f((a+b)/2) < 0
4         b = (a+b)/2;
5     else
6         a = (a+b)/2;
7     end
8 end
9 (a+b)/2
```

Define the function f

```
10 function y = f(x)
11     y = x^3 - 3*x + 1;
12 end
```

ans = 0.3594