

Lecture 2

Monday, January 12, 2026 1:47 AM

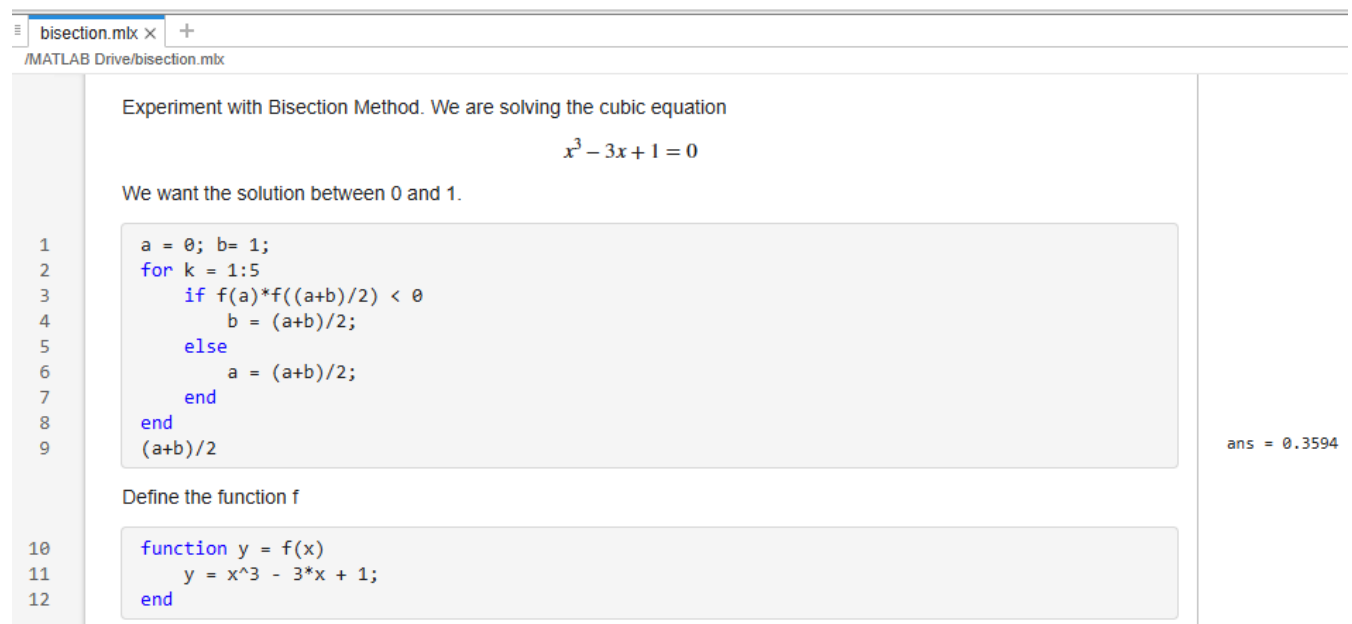
For the *bisection method*, when do we stop the iteration? It depends on the precision we want. Suppose the initial search interval is $[a, b]$. After halving this interval n times, the search interval is of length $\frac{b-a}{2^n}$. By taking the midpoint x_* of this interval as the approximate solution, we know that the error is at most $\frac{b-a}{2^{n+1}}$. That is,

$$|x_* - x_{ex}| \leq \frac{b-a}{2^{n+1}}$$

Therefore, if we want the error to be less than some ϵ , we need to make sure n is large enough such that $\frac{b-a}{2^{n+1}} < \epsilon$.

Another way to terminate the iteration is when we see two consecutive iterations give the same result with a given number of decimal places. For example, suppose we want the result to be correct to 2 decimal places. We stop at x_n (the result at the n 'th iteration) whenever x_{n-1} and x_n have the same 2 digits after the decimal point.

Implement on Matlab: (Live script)



The image shows a MATLAB Live Script titled 'bisection.mlx'. The script is designed to solve the cubic equation $x^3 - 3x + 1 = 0$ using the bisection method. It starts by defining the initial interval $a = 0$ and $b = 1$. A loop runs for 5 iterations, updating the interval boundaries based on the sign of the function at the midpoint. The function $f(x) = x^3 - 3x + 1$ is defined at the bottom. The final result is displayed as `ans = 0.3594`.

```
bisection.mlx × +  
/MATLAB Drive/bisection.mlx  
  
Experiment with Bisection Method. We are solving the cubic equation  

$$x^3 - 3x + 1 = 0$$
  
  
We want the solution between 0 and 1.  
  
1 a = 0; b = 1;  
2 for k = 1:5  
3     if f(a)*f((a+b)/2) < 0  
4         b = (a+b)/2;  
5     else  
6         a = (a+b)/2;  
7     end  
8 end  
9 (a+b)/2  
  
Define the function f  
  
10 function y = f(x)  
11     y = x^3 - 3*x + 1;  
12 end  
  
ans = 0.3594
```