Lecture 20: Simpson's Method for Approximating Integrals (03/04/2026)

The trapezoid rule approximates the function using a straight line. Simpson's method improves this by approximating the function with a quadratic polynomial. Instead of connecting points with straight lines, Simpson's rule fits a parabola through the points.

0.14 Three Point Approximation

Using points

$(x_0, f(x_0)), \quad (x_1, f(x_1)), \quad (x_2, f(x_2))$

we approximate

$\int_{x_0}^{x_2} f(x)\,dx$ with

$$\frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

If n is even and $h = \frac{b-a}{n}$, then

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

Simpson Rule is:

$$S_n = \frac{h}{3}\left[f(x_0) + 4\sum_{\text{odd}} f(x_k) + 2\sum_{\text{even}} f(x_k) + f(x_n)\right]$$

0.15 Error Order

Simpson's rule has error $|I - S_n| = O(h^4)$ which is much more accurate than the trapezoid rule.