Lecture 21: Gauss-Legendre Quadrature Integration Rule (03/06/2026)

Given:

$$\int_a^b f(x)\,dx$$

We want to use as few evaluations of f as possible while obtaining a good estimate for the integral.

0.16 Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx I_n$$

using n sub intervals.

$$f(x_0), f(x_1), \ldots, f(x_n)$$

Error is $O\left(\frac{1}{n^2}\right)$.

$$\int_a^b f(x)\,dx \approx w_0 f(x_0) + w_1 f(x_1) + \cdots + w_n f(x_n)$$

- $x_0, x_1, \ldots, x_n$ are chosen, not necessarily equally spaced.
- f is evaluated at these points.

Before applying the formula, weights and sample points are predetermined (selected points).

0.17 Linearity

$$\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = w_0 f(x_0) + \cdots + w_n f(x_n)$$

$$\int_a^b g(x)\,dx = w_0 g(x_0) + \cdots + w_n g(x_n)$$

Goal

We want this approximation to be exact for as many special classes of functions as possible.

Choose these functions as:

$$1, x, x^2, \ldots, x^{2n+1}$$

If the formula is exact for these, then it is exact for polynomials of degree $\leq 2n+1$.