Lecture 23: Numerical Integration and Lengendre Polynomials (03/11/2026)

$$\int_a^b f(x)\,dx \approx w_1 f(x_1) + \cdots + w_n f(x_n)$$

weights: $w_1, w_2, \ldots, w_n$

sample points: $x_1, x_2, \ldots, x_n$

- $n = 1$: $w_1 = 2$, $x_1 = 0$
- $n = 2$: $w_1 = w_2 = 1$, $x_1 = -\frac{1}{\sqrt{3}}$, $x_2 = \frac{1}{\sqrt{3}}$

0.18 Theorem

The sample points $x_1, x_2, \ldots, x_n$ are the roots of a polynomial $P_n(x)$ defined recursively as follows:

$$P_0(x) = 1, \quad P_1(x) = x$$

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

$P_n(x)$ are Legendre polynomials.

Test

$$n = 1: \quad P_1(x) = x \Rightarrow x_1 = 0$$

$$n = 2:$$