Lecture 24: Numerical Methods for Differential Equations (03/13/2026)

$$y' = f(x, y)$$

$$y(x_0) = y_0$$

$$x_0, x_1, x_2, x_3, \ldots, x_n = x_0 + nh$$

Discretize the x-axis with step size h.

0.19 Two Types of Methods

1) Numerical Differentiation

Apply at $x = x_n$:

$$y'(x_n) = f(x_n, y(x_n))$$

$$y'(x_n) = f(x_n, y_n)$$

2) Numerical Integration

$$y'(x) = f(x, y(x))$$

Integrate on $[x_n, x_{n+1}]$:

$$\int_{x_n}^{x_{n+1}} y'(x)\,dx = \int_{x_n}^{x_{n+1}} f(x, y(x))\,dx$$

$$y(x_{n+1}) - y(x_n)$$

$$y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} f(x, y(x))\,dx$$

0.20 Finite Difference Approximation

$$y'(x_n) \approx \frac{y_{n+1} - y_n}{h}$$

(1st order forward)

$$\frac{y_{n+1} - y_n}{h} = f(x_n, y_n)$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$