Lecture 25: Numerical Methods Continuation (03/16/26)

$$y' = f(x, y), \quad y(x_0) = y_0$$

Let

$$x_0, x_1, x_2, x_3, \dots$$

$$x_n = x_0 + nh \quad \text{(grid points)}$$

$$h = \text{step size}$$

$$y(x_{n+1}) - y(x_n) = \int_{x_n}^{x_{n+1}} f(x, y(x))\, dx$$

Denote:

$$y_{n+1} - y_n = \int_{x_n}^{x_{n+1}} f(x, y(x))\, dx$$

$$y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} f(x, y(x))\, dx$$

0.21 Forward Euler Method

$$\int_{x_n}^{x_{n+1}} f(x, y(x))\, dx \approx \int_{x_n}^{x_{n+1}} f(x_n, y(x_n))\, dx$$

$$= hf(x_n, y(x_n)) = hf(x_n, y_n) = hf_n$$

$$\therefore \quad y_{n+1} = y_n + hf_n$$

0.22 Backward Euler Method

$$\int_{x_n}^{x_{n+1}} f(x, y(x))\, dx \approx \int_{x_n}^{x_{n+1}} f(x_{n+1}, y(x_{n+1}))\, dx$$

$$= hf(x_{n+1}, y(x_{n+1})) = hf(x_{n+1}, y_{n+1}) = hf_{n+1}$$

$$\therefore \quad y_{n+1} = y_n + hf_{n+1}$$

(Extract y_{n+1} from this equation.)