# Lecture 28: Continuation (04/01/26)

## 10.1 Differential Equation

$$y'' - y' + xy = x$$

with:

$$y(0) = 2, \quad y(1) = 3$$

# 11 Finite Difference Approximation

$$y''(x_k) \approx \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2}$$

$$y'(x_k) \approx \frac{y_{k+1} - y_k}{h}$$

# 12 Substitution

$$\frac{y_{k+1} - 2y_k + y_{k-1}}{h^2} - \frac{y_{k+1} - y_k}{h} + x_k y_k = x_k$$

Multiply by $h^2$:

$$y_{k+1} - 2y_k + y_{k-1} - h(y_{k+1} - y_k) + h^2 x_k y_k = h^2 x_k$$

# 13 Final Coefficient Form

$$(1 - h)y_{k+1} + (-2 + h + h^2 x_k)y_k + y_{k-1} = h^2 x_k$$

# 14 Matrix System

$$AY = B$$

where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix}$$

$$B = \begin{bmatrix} d_1 - by_0 \\ d_2 \\ \vdots \\ d_{n-1} - cy_n \end{bmatrix}$$

# 15 Important Note

The matrix $A$ is **tridiagonal**, meaning only three diagonals are nonzero.

# 16 Solution

$$Y = A^{-1}B$$

We solve the differential equation numerically using linear algebra.