Lecture 29: Partial Differential Equations (04/03/26)

16.1 Physical Setup

Consider a rod of length L.

- Temperature depends on position and time
- Let $u(x,t)$ = temperature at position x and time t

17 Boundary Conditions

$$u(0,t) = 0, \quad u(1,t) = 0$$

These conditions mean both ends of the rod are held at zero temperature.

18 Heat Equation (1D)

$$u_t = cu_{xx}$$

Definitions:

- $u_t = \frac{\partial u}{\partial t}$ (change over time)
- $u_{xx} = \frac{\partial^2 u}{\partial x^2}$ (curvature in space)
- c = diffusion constant

19 Understanding Change

We approximate:

$$u(x, t + \Delta t) - u(x,t)$$

and

$$u(x + \Delta x, t) - u(x, t)$$

19.1 Cases

- If left = right $\Rightarrow$ no change
- If left > right $\Rightarrow$ temperature decreases
- If left < right $\Rightarrow$ temperature increases

Heat flows from hotter regions to colder regions.

20 2D Heat Equation

Let:

$$u(x, y, t)$$

Then:

$$u_t = c(u_{xx} + u_{yy})$$

This describes heat flow in a surface or region.

21 Key Idea

The heat equation models diffusion:

- Heat spreads out over time
- Sharp changes smooth out