Lecture 8: Continue Interpolation (01/28/2026)

$$(x_0, y_o), (x_1, y_1), \ldots, (x_n, y_n)$$

The Lagrange polynomial P whose graph passes through these points.

$$P(x) = \underbrace{y_o \ell_o(x) + y_1 \ell_1(x) + \ldots + y_n \ell_m(x)}_{\text{Lagrange fundamental polynomial}}$$

$$\ell_o(x) = \frac{(x - x_1)(x - x_2)\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots(x_0 - x_n)}$$

$$\ell_1(x) = \frac{(x - x_0)(x - x_2)\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\ldots(x_1 - x_n)}$$

$$\ell_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)\ldots(x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)\ldots(x_2 - x_n)}$$

$$\ell_o(x_0) = 1$$

$$\ell_1(x_k) = 0 \quad \text{for } k \neq 0$$

$$\boxed{\ell_k(x_j) = \begin{cases} 1 \text{ if } k = j \\ 0 \text{ if } k \neq j \end{cases}}$$

Example:

Find a polynomial of degree ≤ 2 such that $P(0) = 1, \quad P(1) = 3, \quad P(2) = 4$

$$P(x) = 1\ell_o(x) + 3\ell_1(x) + 4\ell_2(x)$$

$$\ell_o(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} = \frac{1}{2}(x-1)(x-2)$$

$$\ell_1(x) = \frac{(x-0)(x-2)}{(1-0)(1-2)} = -x(x-2)$$

$$\ell_2(x) = \frac{(x-0)(x-1)}{(2-0)(2-1)} = \frac{1}{2}x(x-1)$$

Therefore

$$P(x) = \frac{1}{2}(x-1)(x-2) - 3x(x-2) + 2x(x-1)$$