Lecture 9: Interpolation Continuation (01/30/2026)

Review

$(x_0, y_o), (x_1, y_1), \ldots, (x_n, y_n)$

There is a unique polynomial with degree $\leq n$ that interpolates these n points.

Lagrange gave us a method to find the polynomial.

$$P(x) = y_o\ell_o(x) + y_1\ell_1(x) + \ldots + y_n\ell_n(x)$$

There is also another method to find the same polynomial which is called *Newton Method*

$$P(x) = c_o + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \ldots + c_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

We need to find $c_o, c_1, c_2, \ldots, c_n$.

To find c_o,

Plug $x = x_0$:

$y_o = P(x_0) = c_o$

To find c_1,

Plug $x = x_1$:

$P(x_1) = c_o + c_1(x_1 - x_0)$

$y_1 = P(x_1) = c_o + c_1(x_1 - x_0)$

We get $c_1 = \frac{y_1 - c_o}{x_1 - x_0}$

Example:

Using Newton's method (Newton's divided difference interpolation polynomial) to find the polynomial of degree ≤ 3 that interpolates the data (1,1),(2,1),(3,2),(0,-1).

$$P(x) = c_o + c_1(x-1) + c_2(x-1)(x-2) + c_3(x-1)(x-2)(x-3)$$

$$\text{Plug } x = 1:$$

$$P(1) = c_o, \text{ so } c_o = 1$$

$$\text{Plug } x = 2:$$

$$P(2) = c_o + c_1(2-1) = 1 + c_1$$

$$1 = 1 + c_1, \text{ so } c_1 = 0$$

$$\text{Plug } x = 3:$$

$$P(3) = c_o + c_1(3-1) + c_2(3-1)(3-2)$$

$$2 = 1 + 0(2) + c_2(2)(1)$$

$$2 = 1 + 2c_2, \text{ so } c_2 = \frac{1}{2}$$

$$\text{Plug } x = 0:$$

$$P(0) = c_o + c_1(0-1) + c_2(0-1)(0-2) + c_3(0-1)(0-2)(0-3)$$

$$-1 = 1 + 0(-1) + \frac{1}{2}(2) + c_3(-6), \text{ so } c_3 = \frac{1}{2}$$

$$\text{Therefore, } P(x) = 1 + \frac{1}{2}(x-1)(x-2) + \frac{1}{2}(x-1)(x-2)(x-3)$$