

Midterm II: Some problems for review

The exam will be taken in class (SCB 303) on Friday 3/20. You will do the exam on paper. You can use a pocket calculator of any kind. Phones, laptops, and notecards are not allowed. The instructor will provide scratched papers for you.

Other exam policies:

The proctor may reassign your seat at the beginning or at any time during the exam.

Using a phone or any unauthorized assistance while the exam is in progress, whether inside or outside of the classroom, is prohibited.

If you need to leave the room for any reason, you must first obtain the proctor's permission. If the proctor is not present in the room and you want to leave, you must wait until he/she comes back.

Violation of any of the above policies is considered as cheating and may result in a score of zero.

The topics to be covered are

- Spline interpolation
- Numerical differentiation
- Numerical integration (including error estimate for Trapezoid Rule)
- Forward and backward Euler methods to solve a differential equation

You should review the homework problems, worksheets, quizzes, examples given in the lectures. It is always a good idea to study for the exam with someone.

Some problems to practice:

1) Check if the following function is a spline. If it is so, find the degree of the spline.

$$(a) f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ 2x^2 - 2x + 1, & 1 < x \leq 2 \end{cases}$$

$$(b) f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ 2x^3 - x, & 1 < x \leq 2 \end{cases}$$

2) Set up the equations satisfied by a quadratic spline through three points $(0, 1)$, $(1, 3)$, $(2, 2)$. Solve those equations to obtain such a spline.

3) Suppose x_0, x_1, x_2, \dots are equally spaced points with step size h on the real line, and f is a function with $f(x_k) = y_k$. Determine the type of numerical differentiation (forward/backward/central).

$$(a) f'(x_k) = \frac{y_{k+1} - y_k}{h} + O(h)$$

$$(b) f'(x_k) = \frac{-3y_k + 4y_{k+1} - y_{k+2}}{2h} + O(h^2)$$

$$(c) f'(x_k) = \frac{y_{k+1} - y_{k-1}}{2h} + O(h^2)$$

$$(d) f'(x_k) = \frac{3y_k - 4y_{k-1} + y_{k-2}}{2h} + O(h^2)$$

(e) $f''(x_k) = \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2} + O(h^2)$

4) From the data

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	8.0	8.4	8.8	9.1	9.5	9.8	10.0

compute approximately $f'(1.3)$ using Newton backward difference at order $O(h^2)$.

5) Approximate the integral $\int_1^3 \frac{1}{x} dx$ using

(a) Trapezoid Rule with 4 subintervals.

(b) Simpson Rule with 4 subintervals.

(c) Gauss-Legendre 1-point rule.

(d) Gauss-Legendre 2-point rule.

6) The error estimate of the Trapezoid Rule for integral is $|I_n - \int_a^b f(x)dx| \leq \frac{M(b-a)^3}{12n^2}$ where n is the number of equal subintervals and $M = \max_{[a,b]} |f''|$. Suppose that you compute the integral $\int_0^3 xe^{-2x} dx$ using the Trapezoid Rule with n equal subintervals. How large must n be so that the error is under 10^{-4} ?

7) Solve the differential equation $y' = x + y$ with initial condition $y(0) = 1$ on the interval $[0, 1]$ with step size $h = 0.25$ using

(a) Forward Euler method

(b) Backward Euler method