Worksheet 3/30/2026

$y'(x_k) = \frac{y_{k+1}-y_k}{h} + O(h) = \frac{y_k - y_{k-1}}{h} + O(h) = \frac{y_{k+1}-y_{k-1}}{2h} + O(h)$

$y'(x_k) = \frac{-3y_k+4y_{k+1}-y_{k+2}}{2h} + O(h^2) = \frac{y_{k-2}-4y_{k-1}+3y_k}{2h} + O(h^2) = \frac{y_{k+1}-y_{k-1}}{2h} + O(h^2)$

$y''(x_k) = \frac{y_k-2y_{k+1}+y_{k+2}}{h^2} + O(h) = \frac{y_{k-2}-2y_{k-1}+y_k}{h^2} + O(h) = \frac{y_{k+1}-2y_k+y_{k-1}}{h^2} + O(h)$

$y''(x_k) = \frac{2y_k-5y_{k+1}+4y_{k+2}-y_{k+3}}{h^3} + O(h^2) = \frac{-y_{k-3}+4y_{k-2}-5y_{k+1}+2y_k}{h^3} + O(h^2) = \frac{y_{k+1}-2y_k+y_{k-1}}{h^2} + O(h^2)$

Consider the boundary value problem $y'' - xy' + y = x^2$, $y(0) = 2$, $y(1) = 1$.

1) With nodal points $0 = x_0 < x_1 < \cdots < x_n = 1$ and $y_k = y(x_k)$, find y_0 and y_n.

2) Derive the difference equations at order $O(h)$ for the problem using central forward difference at all interior nodal points.

3) With $h = 0.25$, solve the system to the approximate values for y_1, y_2, y_3.

4) Solve the problem but now use difference equations at order $O(h^2)$ for the problem using forward difference at all interior nodal points and backward difference at x_n.