Final Exam: Some problems for review

The Final Exam of Math 334 will be taken in class (SCB 211) on Wednesday 4/15 from 11 AM to 1 PM. You will do the exam on paper. You can use a pocket calculator of any kind. Phones, laptops, and notecards are not allowed. The instructor will provide scratched papers for you.

Other exam policies:

The proctor may reassign your seat at the beginning or at any time during the exam.

Using a phone or any unauthorized assistance while the exam is in progress, whether inside or outside of the classroom, is prohibited.

If you need to leave the room for any reason, you must first obtain the proctor's permission. If the proctor is not present in the room and you want to leave, you must wait until he/she comes back.

Violation of any of the above policies is considered as cheating and may result in a score of zero.

The textbook sections to be covered are 3.1, 3.2, 3.3, 3.4, 3.5. You should review the homework problems, worksheets, quizzes, examples given in the lectures. It is always a good idea to study for the exam with someone.

Some problems to practice:

1) Solve the system of ODEs $y_1' = y_1 - y_2,\ y_2' = -6y_1 + 2y_2 + x$ by converting it into a single ODE.

2) Two tanks are with salt water. Tank 1 contains 80 liters of water initially containing 4 grams of salt dissolved in it and tank 2 contains 100 liters of water and initially has 8 grams of salt dissolved in it. Salt water with a concentration of 0.5 gram/liter of salt enters tank 1 at a rate of 4 liters/hour. Through a connecting pipe, water flows from tank 2 into tank 1 at a rate of 1 liter/hour, and from tank 1 into tank 2 at the rate of 5 liters/hour through a different pipe. Water is drained out of tank 2 at a rate of 4 liters/hour. Draw a picture, and set up a system of ODEs together with initial conditions that determine the amount of salt in each tank at any given time.

3) Consider the system of ODEs $y_1' = 2y_1 + 2y_2,\ \ y_2' = 5y_1 - y_2$.

(a) Write the system in matrix form.

(b) Diagonalize the matrix.

(c) Write the general solution to the system.

(d) Sketch the phase portrait of the system of ODEs. What kind of equilibrium is the origin (source, sink, saddle point, or none of them)?

(e) Assuming $y_1(0) = 2, y_2(0) = 1$, determine y_1 and y_2.