# Lecture 15: Continuation (02/20/2026)

## 0.5 Method to Solve Second Order Linear Homogeneous ODE

**Step 1:** Solve the quadratic equation $ar^2 + br + c = 0$.

Suppose you get two distinct roots $r_1$ and $r_2$.

So, $y_1 = e^{r_1 x}$ and $y_2 = e^{r_2 x}$ are two linearly independent solutions.

**Step 2:** All solutions are $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$.

If $r_1$ and $r_2$ are complex roots of $ar^2 + br + c = 0$ there is no difference in the method.

You can simplify your answer using the identity,

$$e^{a+ib} = e^a \cos b + ie^a \sin b \quad \text{Euler's Identity}$$

$$(e^{(1+i)x})' = (1+i)e^{(1+i)x}$$

### 0.5.1 Double Roots:

If the characteristic has repeated or double root, multiply the second solution by x to obtain a linearly independent solution.

$$r_1 = r_2 = r$$

First solution:

$$y_1 = e^{rx}$$

Second solution:

$$y_2 = xe^{rx}$$

General solution:

$$y = c_1 e^{rx} + c_2 x e^{rx}$$