Lecture 16: Continuation (02/23/2026)

0.6 Solving Constant Coefficient Equations

Solve the ODE $y'' - 2y' + 5y = 0$. Find $\lim_{x \to \infty} y(x)$ and $\lim_{x \to -\infty} y(x)$

$$y = e^{rx}$$

$$y' = re^{rx}$$

$$y'' = r^2 e^{rx}$$

$$r^2 e^{rx} - 2re^{rx} + 5e^{rx} = 0$$

$$e^{rx}(r^2 - 2r + 5) = 0$$

$$r^2 - 2r + 5 = 0$$

$$r = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$\frac{2 \pm \sqrt{-16}}{2}$$

$$\frac{2 \pm 4i}{2}$$

$$1 \pm 2i$$

$$y_1 = e^{(1+2i)x}, \quad y_2 = e^{(1-2i)x}$$

$$y = e^x(c_1 \cos(2x) + c_2 \sin(2x))$$

$$\lim_{x \to \infty} y(x) = \begin{cases} 0 & \text{if } c_1 \text{ and } c_2 = 0 \\ \text{DNE otherwise} \end{cases}$$

$$\lim_{x \to -\infty} y(x) = 0$$

Suppose the characteristic equation has two complex roots $r = \alpha + \beta i$, then the general solution to the ODE is

$$y = c_1 e^{\alpha x} \cos(\beta x) + c_2 e^{\alpha x} \sin(\beta x)$$

c_1 and c_2 are to be determined by initial condition