Lecture 17: Mechanical Vibration (02/25/2026)

0.7 Second Order Linear ODE Applications

Examples of vibrating systems:

- Pendulum
- Mass-spring system
- Vibrating beam
- Multi-pendulum system

These systems oscillate forces act to restore equilibrium.

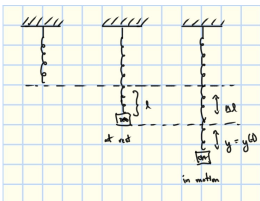

Figure 8: Spring Diagram

<u>Hooke's Law</u>

For a spring, $F = k(\Delta \ell)$ where

- F = force required
- k = stiffness of the spring

- $\Delta\ell$ = amont of stretch

The force is proportional to the stretch.

0.7.1 Vertical Spring - Mass System

Assume the spring itself has **no mass**. A mass m is attached to a vertical spring. The mass stretches the spring until it reaches an **equilibrium position**.

At equilibrium:

$$mg = k(\Delta\ell) \text{ where}$$

- mg = weight of the mass
- $k(\Delta\ell)$ = restoring force of the spring

This determines the equilibrium stretch.

Let $y(t)$ be the displacement from equilibrium.

- $y(t) > 0$: object is <u>below equilibrium</u>
- $y(t) < 0$: object is <u>above equilibrium</u>

Total force on the mass:$F = ma$

Acceleration: $a = y''(t)$

Velocity: $y'(t)$

Position: $y(t)$

Spring force: $y + \Delta\ell$

Thus the spring force becomes $F_s = -k(y + \Delta\ell)$

The negative sign indicates the <u>restoring direction</u>.

0.7.2 Newton's Second Law

$$F = ma = my''$$

Forces acting on the mass:

- Gravity: mg
- Spring force: $F_s = -k(y + \Delta\ell)$

The net force equation is:

$$my'' = mg - k(y + \Delta\ell)$$

$$my'' = mg - k(y + \Delta\ell)$$
$$mg = k(\Delta\ell)$$
$$my'' = -ky$$
$$my'' + ky = 0$$

This is the differential equation for simple harmonic motion. The motion is oscillatory around the equilibrium position.