

Lecture 23: Power Series Representation of a Function (03/11/2026)

- A power series defines a function on the interval of convergence.
- Most functions can be written as a power series.

$$\sum_{k=0}^{\infty} a_k (x - x_0)^k$$

Example:

$$\sum \frac{x^k}{k}$$

$$f(x) = x^2 + 5x + 3 \text{ around } x_0 = 1$$

$$x^2 + 5x + 3 = a_0 + a_1(x - 1) + a_2(x - 1)^2 + \dots$$

Plug $x = 1$:

$$a_0 = 9$$

Differentiate:

$$2x + 5 = a_1 + 2a_2(x - 1) + \dots$$

Plug $x = 1$:

$$a_1 = 7$$

General Formula:

$$f(x) = \sum a_n (x - x_0)^n$$

$$a_n = \frac{f^{(n)}(x_0)}{n!}$$

Taylor Series

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \cdots$$

Example

$$\sin(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$\sin(0) = a_0 = 0$$

$$\cos(x) = a_1 + 2a_2x + 3a_3x^2 + \cdots$$

$$\cos(0) = a_1 = 1$$

$$-\sin(x) = 2a_2 + 6a_3x + \cdots$$

$$-\sin(0) = 2a_2 \Rightarrow a_2 = 0$$