Lecture 24: Power Series Continuation (03/13/2026)

$$f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

$$a_n = \frac{f^{(n)}(x_0)}{n!}$$

Taylor Series:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

Analytic Function:

A function is analytic at x_0 if

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

Example (non-analytic case):

$$f(x) = \begin{cases} e^{-1/x^2}, & -1 < x < 1 \\ 0, & x \le -1 \text{ or } x \ge 1 \end{cases}$$

All derivatives at $x = 0$ are zero, but $f(x)$ is not analytic.

Examples

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\frac{2}{3+5x}$$

$$= \frac{2}{3} \cdot \frac{1}{1+\frac{5}{3}x} = \frac{2}{3}\sum_{n=0}^{\infty}\left(-\frac{5}{3}x\right)^n$$