Lecture 26: System of Differential Equations (03/25/26)

A system of first order ODEs:

$$F_1(y_1, y_2, \ldots, y_n, y_1', y_2', \ldots, y_n', x) = 0$$

$$F_2(y_1, y_2, \ldots, y_n, y_1', y_2', \ldots, y_n', x) = 0$$

Example:

$$\begin{cases} y_1' = y_1 y_2 \\ y_2' = x y_1 + y_2 \end{cases}$$

$$y_1 = y_1(x), \quad y_2 = y_2(x)$$

Second Order Equation as System

$$y'' - 2y' + 3y = x$$

Let:

$$y_1 = y, \quad y_2 = y'$$

Then:

$$y_1' = y_2$$

$$y_2' = 2y_2 - 3y_1 + x$$

0.24 Reduction Method

$$\begin{cases} y_1' = y_1 + y_2 + e^x \\ y_2' = 2y_1 + 3y_2 + x^2 \end{cases}$$

Initial conditions:

$$y_1(0) = 2, \quad y_2(0) = 3$$

Combine equations to form second order ODE:

$$y'' - 3y' + 3y = 0$$

Characteristic equation:

$$r^2 - 3r + 3 = 0$$

$$r = \frac{3 \pm \sqrt{9 - 12}}{2} = \frac{3}{2} \pm \frac{\sqrt{3}}{2} i$$

$$y(x) = e^{\frac{3}{2}x} \left(c_1 \cos\left(\frac{\sqrt{3}}{2} x \right) + c_2 \sin\left(\frac{\sqrt{3}}{2} x \right) \right)$$

Use initial conditions to solve for constants.