Lecture 28: Matrix Inverse Using Row Reduction (04/01/2026)

We compute the inverse of a matrix using augmented matrices.

1.7 Example

Start with:

$$[A \mid I]$$

Perform row operations until:

$$[I \mid A^{-1}]$$

Key Idea: If we can reduce A to the identity matrix, then the matrix is invertible.

1.8 Conclusion

If a matrix has a pivot in every row (no zero rows), then:

$$A \text{ is invertible}$$

2 Eigenvalues and Eigenvectors

2.1 Definition

A scalar λ is an eigenvalue of A if:

$$Av = \lambda v$$

for some nonzero vector v.

2.2 Equivalent Form

$$(A - \lambda I)v = 0$$

2.3 Condition for Nontrivial Solution

$$\det(A - \lambda I) = 0$$

Explanation: This determinant equation is called the **characteristic equation**. Solving it gives eigenvalues.

3 Example Matrix

$$A = \begin{bmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{bmatrix}$$

3.1 Eigenvalues

$$\lambda = 4, -2, 2$$

3.2 Eigenvectors

Solve:

$$(A - \lambda I)v = 0$$

for each eigenvalue.

Key Idea: Eigenvectors describe directions that are unchanged (except scaling) by the matrix transformation.

4 Important Concept

If a matrix has n independent eigenvectors, it can be diagonalized.