Lecture 3

Wednesday, January 14, 2026 11:55 AM

Ordinary differential equation of the form $y' = f(x)$:
The solution is $y = \int f(x)dx$. You can see that there is a constant of integration involved. So, technically there are infinitely many solutions to the differential equation $y' = f(x)$.

If an initial condition is given, say $y(x_0) = y_0$ where x_0 and y_0 are given, then you can use this information to identify the constant of integration that satisfies the initial condition.

Example: $y' = e^{2x}$

Example: $y' = e^{2x},\ y(0) = 3$

Ordinary differential equation of the form $y' = f(y)$:
There is a nice trick for this (the Chain Rule for inverse function).
If viewing $y = y(x)$, we have $y' = \frac{dy}{dx}$. If viewing $x = x(y)$, we have $x' = \frac{dx}{dy}$. So, $x' = \frac{1}{y'} = \frac{1}{f(y)}$.
Then we can write

$$x = \int \frac{1}{f(y)} dy$$

From here, we try to solve back y as a function of x.

Example: $y' = y^2$

[Work on the worksheet]