Lecture 6

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Separable equations

An ODE is separable if we can write it in the form $y' = f(x)g(y)$. Thus, it is a first order ODE.
How do we solve it? Separate y from x as follows:

$$\frac{y'}{g(y)} = f(x)$$

Then integrate both sides with respect to x:

$$\int \frac{y'}{g(y)} dx = \int f(x)dx$$

which is equivalent to

$$\int \frac{dy}{g(y)} = \int f(x)dx$$

This will give us an equation (no longer differential equation) of only y and x. If we can solve for y explicitly in terms of x, we will get an explicit solution. If not, we just leave the equation as is and say that y is given in an implicit form.

Examples:

$y' = xy$ (go through this example carefully)
$y' = \frac{x}{y},\ y(0) = 1$ (go through this example carefully)
$y' = \frac{x}{y^2+1}$(implicit solution only)
$y' = e^{x+y}$ (factor the right-hand side)
$y' = xy + x + y + 1$ (factor the right-hand side)