

Midterm I: Some problems for review

The exam will be taken in class (SCB 211) on Wednesday 2/11. You will do the exam on paper. You can use a pocket calculator of any kind. Phones, laptops, and notecards are not allowed. The instructor will provide scratched papers for you.

Other exam policies:

The proctor may reassign your seat at the beginning or at any time during the exam.

Using a phone or any unauthorized assistance while the exam is in progress, whether inside or outside of the classroom, is prohibited.

If you need to leave the room for any reason, you must first obtain the proctor's permission. If the proctor is not present in the room and you want to leave, you must wait until he/she comes back.

Violation of any of the above policies is considered as cheating and may result in a score of zero.

The textbook sections to be covered are 0.2, 0.3, 1.1 - 1.8. You should review the homework problems, worksheets, quizzes, examples given in the lectures. It is always a good idea to study for the exam with someone.

Some problems to practice:

1) Determine if each of the following equation is an exact differential equation. If so, solve the equation. You may leave the solution in implicit form if an explicit form is difficult to obtain.

a) $(2x + y)dx + (x - 4y)dy = 0$

b) $\frac{1}{y}dx + 3ydy = 0$

c) $\cos x + ye^{xy} + xe^{xy}y' = 0$

2) A container initially has 20 L of seawater with salt concentration 35 g/L. A flow with salt concentration 10 g/L is pumped into the container at a volume rate of 2 L/min. At the same time, well-mixed liquid in the container is pumped out at the rate of 2 L/min. How much time will elapse before the salt concentration in the container is equal to 25 g/L?

3) Let $y = y(x)$ be a solution to the differential equation $y' = \frac{y-y^3}{1+y^2}$.

a) Draw a phase diagram and sketch several representative solutions.

b) Suppose $y(-2) = 1.5$. Find $\lim_{x \rightarrow \infty} y(x)$ and $\lim_{x \rightarrow -\infty} y(x)$.

4) Solve the initial value problem $x(y' + 2) = y$, $y(1) = 2$.

5) Solve the initial value problem $y^2y' = y^3 - 3x$, $y(0) = 1$.