Worksheet 3/30/2026

1) Write the following initial value problems in matrix form $x' = Ax$ with suitable initial conditions.

(a) $\begin{cases} x_1' &= x_1 + x_2 \\ x_2' &= -x_1 + 2x_2 \end{cases}, x_1(0) = 1, x_2(0) = 2$

(b) $\begin{cases} x_1' + 2x_1 &= x_2 \\ x_2' + x_2 &= -x_1 \end{cases}, x_1(1) = -1, x_2(1) = 0$

(c) $\begin{cases} tx_1' + t^2 x_2 + \sin t &= 0 \\ x_2' + e^t x_1 &= t \end{cases}, x_1(0) = 2, x_2(0) = 1$

2) Convert the following ODEs into a linear system of equations with suitable initial conditions.

(a) $y'' + 2y' + y = x, y(0) = y'(0) = 1$

(b) $y'' + xy = x^2, y(1) = 0, y'(1) = 2$

(c) $y''' - 3y'' + 2y = e^x, y(0) = 1, y'(0) = 1$

3) Solve the system of linear equations using inverse of a matrix

$$\begin{cases} 2x_1 + 3x_2 + x_3 = 1 \\ 3x_1 + 3x_2 + x_3 = 2 \\ 2x_1 + 4x_2 + x_3 = 3 \end{cases}$$

4) Diagonalize the following matrix. That is to write it in the form $A = PDP^{-1}$ where P is an invertible matrix and D is a diagonal matrix.

(a) $\begin{bmatrix} 2 & 7 \\ 8 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{bmatrix}$