Worksheet 3/9/2026

1) Reindex the following power series to have that power x^k.

$(a)\ \sum_{k=3}^{100} \frac{1}{k} x^{k-2}, \qquad (b)\ \sum_{k=3}^{100} k!\, x^{k+1}, \qquad (c)\ \sum_{k=m}^{\infty} a_k x^{k-1}, \qquad (d)\ \sum_{k=m}^{\infty} a_{k+1} x^{k-m}$

2) Find the radius of convergence and interval of convergence of the following power series.

(a) $\sum_{k=1}^{\infty} \frac{x^k}{k}$

(b) $\sum_{k=1}^{\infty} \frac{2^k (x-1)^{2k}}{k}$

3) Express the following functions as a power series.

(a) $f(x) = \frac{1}{2+x}$ about $x_0 = 0$.

(b) $f(x) = \frac{1}{2-3x}$ about $x_0 = 0$.

(c) $f(x) = \frac{5}{2+3x^2}$ about $x_0 = 0$.

(d) $f(x) = \frac{1}{1-x}$ about $x_0 = 3$.

(e) $f(x) = \frac{1}{1-x^2}$ about $x_0 = 2$.

(f) $f(x) = x^2 + x - 3$ about $x_0 = 1$.

(g) $f(x) = x^3 + x - 3$ about $x_0 = 1$.