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**Math 213**  
**Exam I**  
February 2020

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Instructor: Department Exam

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**Instructions**

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
- C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 4 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Please do not talk about the test with other students until after the last day of the exam.



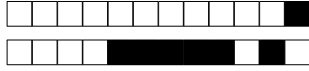
**Part I: Multiple Choice Questions:** (*4 points each*) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** mark in the other boxes.

1 ♣ For vectors  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  in  $\mathbb{R}^n$  ( $n \geq 2$ ), and  $c$  a real scalar, which of the following are valid, well-defined expressions? Mark all that apply?

- $\mathbf{u} \cdot \mathbf{v} + \mathbf{w}$ .
- $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$ .
- $\|\mathbf{u} \cdot \mathbf{v}\|$ .
- $(c\mathbf{u} - \mathbf{w}) \cdot \mathbf{v}$ .
- $c(\mathbf{u} + \mathbf{w})$ .

2 If  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal vectors then what can we say about the projection of  $\mathbf{u}$  onto  $\mathbf{v}$ ?

- The projection will not be orthogonal to  $\mathbf{v}$ .
- The projection is the zero vector.
- The projection will be longer than  $\mathbf{v}$ .
- The projection will not be orthogonal to  $\mathbf{u}$ .
- The projection will be longer than  $\mathbf{u}$ .
- We can't conclude anything from the information given.



3 Which of the following are orthogonal to the vector  $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ .

$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ .

$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ .

$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ .

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ .

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4 ♣ Which of the following matrices is in reduced row echelon form (RREF). Mark all that apply.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

5 ♣ Given that a system of equations has augmented matrix  $[A|\mathbf{b}]$  that row reduces to the row echelon form  $\left[ \begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 3 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$ . Which of the following must be true about the system. Mark all that apply.

- The rank of  $A$  is 3.
- The system is inconsistent.
- The columns of  $A$  span all of  $\mathbb{R}^3$
- The system has a unique solution.
- The system has infinitely many solutions.
- The columns of  $A$  are linearly independent.
- The system has 1 free variable.



6 ♣ Which of the following sets of vectors is linearly independent? Mark all that apply.

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \right\}$

$\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

7 Express the vector  $\mathbf{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$  as a linear combination

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$c_1 = 2, c_2 = -3$

$c_1 = -1, c_2 = 2$

$c_1 = 3, c_2 = -2$

$c_1 = 3, c_2 = 2$

  $\mathbf{b}$  is not a linear combination of these vectors

$c_1 = 2, c_2 = -1$

$c_1 = -2, c_2 = 1$

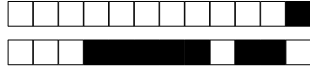


8 ♣ Let  $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$ , and let  $A$  be the  $3 \times 2$  matrix with  $\mathbf{u}$  and  $\mathbf{v}$  as its columns. Which of the following must be true? Mark all that apply.

- If  $\mathbf{b} \in \text{span}(\mathbf{u}, \mathbf{v})$ , then  $A\mathbf{x} = \mathbf{b}$  has a solution.
- $\text{span}(\mathbf{u}, \mathbf{v})$  is a plane.
- $\text{span}(\mathbf{u}, \mathbf{v})$  is NOT all of  $\mathbb{R}^3$ .
- If  $\mathbf{b} \in \text{span}(\mathbf{u}, \mathbf{v})$ , then  $\{\mathbf{u}, \mathbf{v}, \mathbf{b}\}$  is linearly dependent.
- If at least one of  $\mathbf{u}$  and  $\mathbf{v}$  is not the zero vector, then  $\text{span}(\mathbf{u}, \mathbf{v})$  contains infinitely many vectors.
- If  $\text{rank}(A) = 2$ , then  $\mathbf{u} = c\mathbf{v}$  for some constant  $c$ .

9 ♣ Let  $A$  be an  $n \times n$  matrix. Which of the following facts are equivalent to  $A$  being invertible? Mark all that apply.

- $A$  has at least one positive entry.
- There is an  $n \times n$  matrix  $C$  such that  $AC$  is the  $n \times n$  identity matrix.
- The columns of  $A$  span  $\mathbb{R}^n$ .
- $\text{rank } A = n$
- $A$  commutes with  $B$  for all  $n \times n$  matrices  $B$ .
- $A$  has a pivot in every row.



10 ♣ Which of the following matrices are invertible? Mark all that apply.

$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 4 & 4 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

11 ♣ Which of the following are elementary matrices? Mark all that apply.

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 15 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



12 ♣ Let  $A, B,$  and  $C$  be invertible  $n \times n$  matrices. Which of the following must be true? Mark all that apply.

- $A^T B^T$  is invertible.
- $(A^{-1})^T = (A^T)^{-1}$
- $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$
- $((AB)^{-1})^T = (B^{-1})^T (A^{-1})^T$
- If  $AB = AC$  then  $B = C$ .
- $(ABC)^T = A^T B^T C^T$
- $(AB)^{-1} = B^{-1} A^{-1}$

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**Part II: Short Answer Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

a) True or False: A homogeneous system of equations must be consistent.

b) True or False: The vectors  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  span all of  $\mathbb{R}^3$ .

c) How many solutions are there to a system of linear equations with augmented matrix that row reduces to  $\left[ \begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 3 \end{array} \right]$ ? \_\_\_\_\_

d) Write the following system of equation

$$\begin{aligned} 3x - y + 2z &= 7 \\ x + 2y - z &= 13 \\ 2x + y + z &= 1 \\ y + z &= -3 \end{aligned}$$

as a matrix-vector equation  $A\mathbf{x} = \mathbf{b}$  (not as an augmented matrix):

e) If  $A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ , then  $A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$ .



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 0  1  2  3  4  5  6  7  8  9  10  11  12 *Administrative Use Only*

Fill in the blank with the appropriate answer. 2 points per answer unless stated otherwise.

a) Let

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}.$$

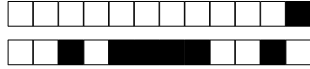
Compute the following matrix products (if the product is not defined write *not defined* in the space provided).

$$AB = \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix}, \quad AC = \begin{bmatrix} \phantom{0} \\ \phantom{0} \\ \phantom{0} \end{bmatrix}, \quad CA = \begin{bmatrix} \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} \end{bmatrix}.$$

b) Given that  $\mathbf{x} = \mathbf{u} + 3\mathbf{v}$  where  $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ , find the value of  $\mathbf{x} \cdot \mathbf{u}$ .

c) Find the equation for the line passing through the point  $P = (0, 1)$  and with normal vector  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ .

d) Find the projection of  $\mathbf{v}$  onto  $\mathbf{u}$  where  $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$ .



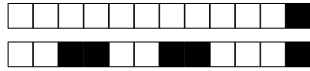
**Part III: Free Response Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

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0 1 2 3 4 5 6 *Administrative Use Only*

Geometrically show what is meant by subtracting the two vectors  $\mathbf{x} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$  and  $\mathbf{y} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$  from each other, i.e. plot  $\mathbf{x} - \mathbf{y}$  AND  $\mathbf{y} - \mathbf{x}$  on separate axes.

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0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Solve the system of equations. If it is inconsistent, state so. If there are infinitely many solutions, express them in parametric-vector form.

$$\begin{aligned}x - 2y - z &= 1 \\2x + y + 3z &= 12 \\3x - 4y - z &= 7\end{aligned}$$

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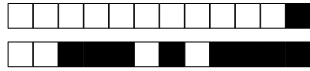
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0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 1 & 6 \\ -1 & 0 & 3 \end{bmatrix}.$$

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0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Consider the matrices

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \text{and} \quad A_3 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Is the set  $\{A_1, A_2, A_3\}$  linearly independent? Justify your answer.

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