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001

Math 213
Exam I
October 2019

Name: _____

Section: _____

Instructor: Department Exam

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Instructions

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
- C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 4 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Please do not talk about the test with other students until after the last day of the exam.



Part I: Multiple Choice Questions: (4 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** mark in the other boxes.

1 Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 4 & 5 & -3 \\ -2 & -1 & -4 & -5 & 4 \\ -5 & -3 & -11 & -14 & 9 \end{bmatrix}.$$

Which of the following matrices is the reduced row echelon form of A ?

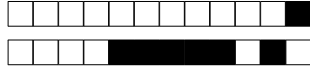
$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$



2 ♣ Which of the augmented matrices will correspond to systems of linear equations with more than one solution? Mark all that apply.

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 \end{array} \right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 \end{array} \right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \end{array} \right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -3 \end{array} \right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 1 & 1 & -3 & 0 \end{array} \right]$

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3 ♣ Which of the following sets of vectors are linearly independent? Mark all that apply.

- $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

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4 ♣ Suppose that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{b} \in \mathbb{R}^5$, and that the system of equations corresponding to the augmented matrix

$$[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \mid \mathbf{b}]$$

is consistent. Which of the following *must* be true? Mark all that apply.

- There are scalars c_1, c_2, c_3 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{b}$.
- $\text{Span} \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} = \mathbb{R}^5$.
- $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{b} \}$ is linearly dependent.
- $\{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \}$ is linearly independent.
- The rank of $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$ is equal to 3.
- $\mathbf{b} \in \text{Span} \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \}$.
- The solution to $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \mid \mathbf{b}]$ is unique.

5 The points

$$A = (1, 2, 3), B = (2, 3, 5), C = (6, 4, -1), D = (1, 2, -1) \in \mathbb{R}^3$$

form the corners of a four-sided figure in \mathbb{R}^3 whose sides give the vectors $\vec{AB}, \vec{BC}, \vec{CD}, \vec{DA}$. At which of the points A, B, C, D are the vectors which meet at the corner orthogonal?

- D
- C
- B
- A
- none of them

6 Let $\mathbf{v}_1 = [1, 2, 3], \mathbf{v}_2 = [2, 3, 5], \mathbf{v}_3 = [6, 4, -1], \mathbf{v}_4 = [1, 2, -1] \in \mathbb{R}^3$ be vectors. Let $\mathbf{v} = [x, y, z]$. Solve the vector equation for \mathbf{v} :

$$2\mathbf{v}_1 - 4\mathbf{v}_2 + \mathbf{v} = 7\mathbf{v}_3 - \mathbf{v}_4.$$

- $[-47, -34, -8]$
- $[-47, -34, 8]$
- $[47, 34, 8]$
- $[47, -34, 8]$
- $[47, 34, -8]$
- $[-47, 34, -8]$



7 ♣ If $\mathbf{v}_1 = [1, 1, 3]$, $\mathbf{v}_2 = [2, 0, 6]$, which of the following are linear combinations of $\mathbf{v}_1, \mathbf{v}_2$?

- $[1, -1, 5]$
- $[-3, -3, -9]$
- $[3, 1, 8]$
- $[1, 3, 3]$
- $[2, 0, 6]$
- $[0, 0, 0]$

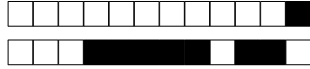
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Let $\mathbf{v} = [0, -1]$, $\mathbf{w} = [\sqrt{3}, -1]$ and find the angle between \mathbf{v} and \mathbf{w} .

- $2\pi/3$
- $\pi/4$
- $\pi/2$
- 0
- $\pi/3$
- $\pi/6$

9 Let A, B , and C be $n \times n$ matrices and r a scalar. Which of the following properties of matrix operations does *not* always hold?

- $r(A + B) = rA + rB$
- $A + B = B + A$
- $AB = BA$
- $(rA)B = A(rB)$
- $(AB)C = A(BC)$
- $A(B + C) = AB + AC$



10 ♣ Which of the following statements is always true? Mark all that apply.

- If the columns of an $m \times n$ matrix span \mathbb{R}^m then the columns of A are linearly independent.
- If the matrix B^T is invertible then B is invertible.
- If A is $n \times n$ and invertible, then $\text{rank}(A) = n$.
- If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{b}$ has a solution for every \mathbf{b} in \mathbb{R}^n then $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution.
- If A is an $m \times n$ matrix with $m > n$ then A is invertible.
- If A is an $n \times n$ matrix and $AB = I_n$, then $BA = I_n$.

11 ♣ Let A be an $m \times n$ matrix. Which of the following must be true? (Mark all that apply.)

- A^T is $n \times m$.
- $A + A^T$ is defined and symmetric.
- $(A^T)^T = A$.
- None of these is correct.
- AA^T is symmetric.
- A^T is symmetric.



12 Find the inverse of the following elementary matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

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Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 0 1 2 3 4 5 6 7 8 9 10 11 12 *Administrative Use Only*

Fill in the blank with the appropriate answer. 2 points per answer.

- a) Find the projection of the point $(1, 2)$ onto the line with equation $y = -x$.

- b) Consider the lines

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

These lines intersect at the point $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$.

- c) The reduced row echelon form of the matrix $\begin{bmatrix} 1 & 0 & 3 & -2 \\ -3 & 1 & -8 & 6 \\ 1 & -1 & 2 & -2 \end{bmatrix}$ is

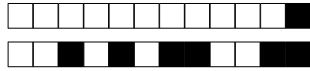
$$\begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}.$$

- d) Define what it means for the set of vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ to be linearly independent:

_____.

- e) If A and B are invertible, then $(AB)^{-1} = \underline{\hspace{2cm}}$.

- f) True or False: If A is a 2×2 matrix and $A^2 = 0$ then $A = 0$.



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 0 1 2 3 4 5 6 7 8 9 10 11 12 *Administrative Use Only*

Fill in the blank with the appropriate answer. 2 points per answer unless stated otherwise.

- a) Let $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$. If $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix}$, find the value of k so that $\mathbf{b} \in \text{Span}\{\mathbf{v}, \mathbf{u}\}$.

$$k = \underline{\hspace{2cm}}$$

- b) (4 points) If A is a 5×4 matrix and B is a 4×3 matrix and C a 3×6 matrix, for each of the following products, state the size (dimensions) of the product, or if the product is not defined, state that it is undefined.

$$AB: \underline{\hspace{2cm}} \quad BA: \underline{\hspace{2cm}} \quad AC: \underline{\hspace{2cm}} \quad BC: \underline{\hspace{2cm}}$$

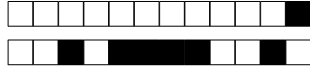
- c) Find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. $\underline{\hspace{2cm}}$

- d) Write the following system as a matrix equation $A\mathbf{x} = \mathbf{b}$

$$3x_1 - 3x_2 - x_3 = 0, \quad 2x_2 - 4x_3 + 5x_4 + 6 = 0$$

- e) Compute the matrix product

$$\begin{bmatrix} -3 & -4 & 6 \\ 0 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & -1 \end{bmatrix} = \underline{\hspace{2cm}}$$



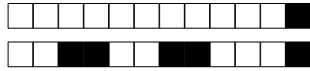
Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

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0 1 2 3 4 5 6 7 *Administrative Use Only*

If $\mathbf{u} \in \mathbb{R}^n$ is orthogonal to $\mathbf{v} \in \mathbb{R}^n$ and $\mathbf{w} \in \mathbb{R}^n$, show that \mathbf{u} is orthogonal to $2\mathbf{v} - 8\mathbf{w}$.

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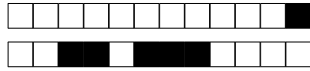
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Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ be non-zero vectors that are orthogonal. Explain why

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \mathbf{0}.$$

(*Hint: this should be a precise explanation using the formula for a projection.*)

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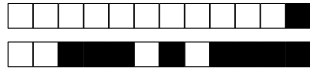
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Consider the planes in \mathbb{R}^3 defined by the equations

$$-x - 2z = 3 \quad \text{and} \quad 2x + y + 7z = -7.$$

Find the *vector form* equation of the line of intersection between these two planes.

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0 1 2 3 4 5 6 7 *Administrative Use Only*

Find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

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