

NS



001

Math 213
Final Exam
December, 2019

Name: _____

Section: _____

Instructor: _____

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Instructions

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Write your name, section number, and instructor in the space provided, and **COMPLETELY FILL IN** the correct boxes for your BYU ID and for the correct answers in the multiple choice section.
- C) Multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 3 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Do not talk about the test with other students until after the exam period is over.



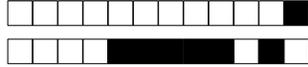
Part I: Multiple Choice Questions: (3 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** leave any marks in the other boxes.

1 ♣ Suppose that A is a 7×5 matrix with linearly independent columns. Which of the following statements must be true? Mark all that apply.

- The equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for all $\mathbf{b} \in \mathbb{R}^7$.
- $\text{rank}(A) = 5$
- A has 5 pivots.
- The columns of A span \mathbb{R}^7 .
- The system represented by the augmented matrix $[A | \mathbf{b}]$ is consistent for all $\mathbf{b} \in \mathbb{R}^7$.
- The equation $A\mathbf{x} = \mathbf{0}$ has a unique solution.
- $\text{Nullity}(A) = 2$

2 ♣ Suppose that U is a $n \times n$ matrix with orthonormal columns. Which of the following must be true? Mark all that apply.

- $\text{Nullity}(A) = n$
- U has n nonzero singular values.
- U is orthogonally diagonalizable.
- U is invertible, with $U^{-1} = U^T$.
- U has an eigenvector \mathbf{v} with eigenvalue $\lambda = 1$.
- $\|U\mathbf{x}\| = \|\mathbf{x}\|$ for all $\mathbf{x} \in \mathbb{R}^n$.
- $\text{rank}(A) = n$
- U has orthonormal rows.



3 Find the singular values of the following matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}.$$

- 1, -1
- 4, 2
- 2, $\sqrt{2}$, 0
- 1, -1, 0
- 2, $\sqrt{2}$
- 4, 2, 0

4 ♣ Let A be a 4×7 matrix with $\text{rank}(A) = 3$. Which of the following must be true? Mark all that apply.

- $\dim(\text{Null}(A)^\perp) = 4$
- 0 is a singular value of A .
- $\dim(\text{Row}(A)^\perp) = 4$
- $\dim(\text{Row}(A)) = 4$
- $\dim(\text{Null}(A)) = 1$
- $\dim(\text{Col}(A)) = 3$



5 Let A be a 5×7 matrix with rank 4. What is the dimension of the null space of A ?

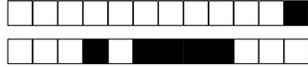
- 3
- 4
- 5
- 1
- 0
- 2

6 Let

$$C = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$$

Find $x \in \mathbb{R}$ such that $C^2 = \mathbf{0}$:

- 1
- 2
- 0
- 1
- 2
- 3



7 Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation that rotates \mathbf{x} counterclockwise by $\pi/2$ radians. What is the standard matrix $[T]$?

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

8 Let A , B , and C be $n \times n$ matrices. The property $A(BC) = (AB)C$ is called.

commutative

additive identity

associative

multiplicative identity

left distributivity

right distributive



9 Solve

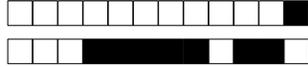
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 2 & -14 \\ 6 & -28 \end{bmatrix}$$

for $a, b \in \mathbb{R}$:

- $a = -2, b = -7.$
- $a = 7, b = -2.$
- $a = -7, b = -2.$
- $a = 2, b = 7.$
- $a = -7, b = 2.$
- $a = -2, b = 7.$
- $a = 2, b = -7.$

10 ♣ Let A be an invertible square matrix with eigenvalue λ and corresponding eigenvector \mathbf{x} . Which of the following must be true?

- $A\mathbf{x} = \lambda\mathbf{x}$
- $A^{-1}\mathbf{x} = \frac{1}{\lambda}\mathbf{x}$
- $\det(A - \lambda I) = 0$
- $\mathbf{x} = \mathbf{0}$
- $A - \lambda I$ is invertible



11 ♣ Which of the following matrices are orthogonally diagonalizable? Mark all that apply.

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 4 & 0 \\ -1 & 0 & -3 \end{bmatrix}$

12 Compute the characteristic polynomial of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{bmatrix}$.

$-\lambda^3 + 4\lambda^2 - 5\lambda$

$-\lambda^3 + 3\lambda^2 - 3\lambda + 1$

$-\lambda^3 + 4\lambda^2 + 3\lambda$

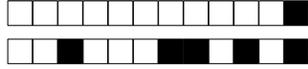
$-\lambda^3 - 3\lambda^2 + 5\lambda + 2$

$-\lambda^3 - 3\lambda^2 - 5\lambda - 1$

$-\lambda^3 - 4\lambda^2 - 5\lambda - 2$

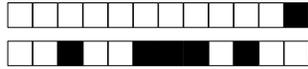
$-\lambda^3 + 4\lambda^2 - 5\lambda + 2$

None of the above



13 ♣ Let A be a 3×3 real matrix with $A^T = A$ and eigenvalues $2, 2, -1$ with eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ respectively. Which of the following must be true? Mark all that apply.

- A is diagonal.
- $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$
- A is invertible.
- A is orthogonal.
- There is an orthogonal Q such that $Q^T A Q$ is diagonal.
- $\mathbf{v}_1 \cdot \mathbf{v}_3 = 0$
- The geometric multiplicity of $\lambda = 2$ is two.



Part II: Short Answer Questions: Write your answers to the questions below in the space provided.

14

0 1 2 3 4 5 6 7 8 9 10 11 12 *Administrative Use Only*

- a) Let W be the subspace of \mathbb{R}^4 spanned by the vectors $\mathbf{v}_1 = [1 \ 1 \ 0 \ -1]^T$ and $\mathbf{v}_2 = [0 \ 0 \ 1 \ 4]^T$. Find a basis for W^\perp .

$$\left\{ \begin{array}{l} \\ \\ \\ \end{array} \right\}$$

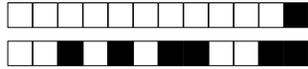
- b) Let W be the subspace of \mathbb{R}^4 spanned by the vectors $\mathbf{v}_1 = [2 \ 1 \ 0 \ 1]^T$ and $\mathbf{v}_2 = [0 \ -1 \ 1 \ 1]^T$, and let $\mathbf{y} = [1 \ 2 \ 1 \ 2]^T$. Find the projection of \mathbf{y} onto W and the perpendicular component of \mathbf{y} to W .

$$\text{proj}_W \mathbf{y} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \quad \text{and} \quad \text{perp}_W \mathbf{y} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

- c) State the precise definition of the following: The vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ span a subspace W if

- d) If A is a 3×3 matrix with $\det A = -3$, then $\det(2A) =$ _____.

- e) If $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 6$, then $\det \begin{bmatrix} d & e & f \\ 3a & 3b & 3c \\ g & h & i \end{bmatrix} =$ _____.



15

0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

a) Suppose that A is a 3×3 invertible matrix that is upper triangular and whose entry a_{11} in the first row and first column is equal to -1 . What is the entry in the first row and first column of A^{-1} ? _____

b) State the precise definition of a linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$.

c) State the precise definition of an eigenvalue for a matrix A .

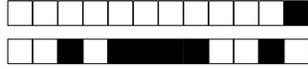
d) If A is a 3×3 matrix with eigenvalues $1, 0, -1$ and corresponding eigenvectors

$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ respectively, and if $\mathbf{x} = -2\mathbf{u} - \mathbf{v} + 3\mathbf{w}$, then

$$A^{1001}\mathbf{x} = \begin{bmatrix} \\ \\ \end{bmatrix}.$$

e) Write the Spectral Decomposition of the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$:

$A =$ _____



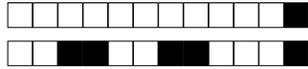
Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

16

0 1 2 3 4 5 6 7 *Administrative Use Only*

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}.$$



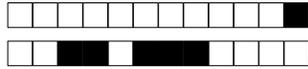
17

0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Let

$$B = \begin{bmatrix} 1 & 1 & 3 & 1 & -2 \\ -3 & 1 & -5 & 0 & -4 \\ -1 & 1 & -1 & 1 & -4 \end{bmatrix}.$$

Find bases for each of the following subspaces: $\text{Row}(B)$, $\text{Col}(B)$, and $\text{Null}(B)$. Clearly label which subspace each basis belongs to.



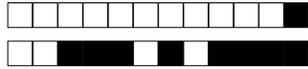
18

0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Orthogonally diagonalize the following matrix

$$A = \begin{bmatrix} 3 & -3 \\ -3 & -5 \end{bmatrix}$$

(i.e. find a diagonal matrix D and an orthogonal matrix Q such that $A = QDQ^T$).



19

0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Find a QR-decomposition of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$



20

0 1 2 3 4 5 6 7 8 *Administrative Use Only*

Find a singular value decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}.$$