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001

**Math 213**  
**Practice Final**  
December, 2019

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Instructor: \_\_\_\_\_

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**Instructions**

- A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- B) Write your name, section number, and instructor in the space provided, and **COMPLETELY FILL IN** the correct boxes for your BYU ID and for the correct answers in the multiple choice section.
- C) Multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
- D) Multiple choice questions are worth 3 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
- E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
- F) No books, notes, or calculators are allowed.
- G) Do not talk about the test with other students until after the exam period is over.



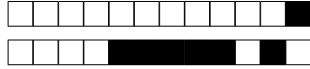
**Part I: Multiple Choice Questions:** (3 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but **DO NOT** leave any marks in the other boxes.

1 ♣ Suppose  $A$  is a  $3 \times 4$  matrix with a pivot in each row. Which of the following must be true? Mark all that apply.

- The equation  $A\mathbf{x} = \mathbf{b}$  has infinitely many solutions for each  $\mathbf{b} \in \mathbb{R}^3$ .
- The columns of  $A$  are linearly independent.
- The equation  $A\mathbf{x} = \mathbf{0}$  has a unique solution.
- There is a vector  $\mathbf{b} \in \mathbb{R}^3$  for which  $A\mathbf{x} = \mathbf{b}$  has no solution.
- $\text{Nullity}(A) = 1$
- The columns of  $A$  span  $\mathbb{R}^3$ .
- $\text{rank}(A) = 4$

2 ♣ Suppose that  $U$  is a  $n \times n$  orthogonal matrix. Which of the following must be true? Mark all that apply.

- $U$  has  $n$  distinct eigenvalues.
- The columns of  $U$  are linearly independent.
- Every eigenvector of  $U$  has length 1.
- $\text{rank}(A) = n$
- $\det U = 1$
- $U^T U$  is the identity matrix.
- $(U\mathbf{x}) \cdot (U\mathbf{y}) = \mathbf{x} \cdot \mathbf{y}$  for all  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ .
- $U$  has orthonormal columns.



- 3 Find the singular values of the following matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

- 5, 3, 1
- $\sqrt{7}, \sqrt{2}$
- $\sqrt{5}, \sqrt{3}, 1$
- 1, -1, 0
- 7, 2, 0
- $\sqrt{7}, \sqrt{2}, 0$

- 4 Performing the Gram-Schmidt procedure on the vectors

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

produces which of the following sets? (Only select one of the following choices.)

- $\{[1 \ 1 \ 2 \ 1]^T, [1 \ 1 \ -1 \ 0]^T, [-1 \ 1 \ 0 \ 0]^T\}$
- $\{[1 \ 1 \ 2 \ 1]^T, [-\frac{1}{3} \ 0 \ -\frac{1}{3} \ 1]^T, [-\frac{22}{21} \ \frac{20}{21} \ -\frac{2}{21} \ \frac{2}{7}]^T\}$
- $\{[1 \ 1 \ 2 \ 1]^T, [-1 \ 1 \ 0 \ 0]^T, [-\frac{1}{6} \ -\frac{1}{6} \ -\frac{1}{3} \ 1]^T\}$
- $\{[1 \ 1 \ 2 \ 1]^T, [1 \ 1 \ -1 \ 0]^T, [-\frac{1}{3} \ 0 \ -\frac{1}{3} \ 1]^T\}$
- $\{[1 \ 1 \ 2 \ 1]^T, [1 \ 1 \ -1 \ 0]^T, [-\frac{22}{21} \ \frac{20}{21} \ -\frac{2}{21} \ \frac{2}{7}]^T\}$
- $\{[1 \ 1 \ 2 \ 1]^T, [-1 \ 1 \ 0 \ 0]^T, [-\frac{22}{21} \ \frac{20}{21} \ -\frac{2}{21} \ \frac{2}{7}]^T\}$



5 ♣ Let  $A$  be a  $5 \times 17$  matrix with rank 4. What is the dimension of the null space of  $A$ :

- 3
- 4
- greater than 4
- 1
- 0
- 2

6 Let

$$C = \begin{bmatrix} 1 & 1 & 1 \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}$$

Find  $x \in \mathbb{R}$  such that  $C^2 = \mathbf{0}$ :

- 1
- 2
- 0
- 1
- 2
- 3



7 Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation that reflects  $\mathbf{x}$  across the  $y$ -axis. What is the standard matrix  $[T]$ ?

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

8 Let  $A$ ,  $B$ , and  $C$  be  $n \times n$  matrices. The property  $A(B + C) = AB + AC$  is called.

commutative

additive identity

associative

multiplicative identity

left distributive

right distributive

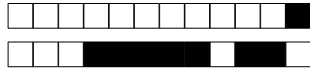


9 ♣ If  $A^{-1} = A^T$ , then the matrix  $A$  is always (mark all that apply):

- the zero matrix
- the identity matrix
- orthogonal
- symmetric
- triangular
- square
- diagonal
- invertible

10 Let  $\mathbf{u}_1 = [1, 1]^T$ ,  $\mathbf{u}_2 = [1, -1]^T$ ,  $\mathbf{y} = [3, 5]^T$ . Find the coordinates of  $\mathbf{y}$  relative to the basis  $\{\mathbf{u}_1, \mathbf{u}_2\}$ .

- $[-1, -4]^T$
- $[-1, 4]^T$
- $[4, -1]^T$
- $[1, 4]^T$
- $[1, -4]^T$
- $[4, 1]^T$
- $[-4, -1]^T$
- $[-4, 1]^T$



11 ♣ Let  $A$  be a square matrix with eigenvalue  $\lambda$  and corresponding eigenvector  $\mathbf{x}$ . Which of the following must be true?

- $2\mathbf{x}$  is an eigenvector of  $A$  with eigenvalue  $2\lambda$
- $\lambda$  is an eigenvalue of  $A^T$
- $\lambda \neq 0$
- $A^3\mathbf{x} = \lambda^3\mathbf{x}$
- $\mathbf{x} \in \text{Nul}(A - \lambda I)$

12 ♣ Which of the following matrices are orthogonally diagonalizable? Mark all that apply.

- $\begin{bmatrix} 1 & -2 & 4 \\ -2 & 4 & 5 \\ 4 & 5 & 3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & -6 \\ 0 & 0 & 5 \end{bmatrix}$
- $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$
- $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$
- $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
- $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 5 & 1 \\ -1 & 1 & -3 \end{bmatrix}$



13 Compute the characteristic polynomial of  $\begin{bmatrix} 3 & -2 & 0 \\ 4 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

- $-\lambda^3 + 3\lambda^2 - 7\lambda + 5$
- $-\lambda^3 + 3\lambda^2 - 7\lambda$
- $-\lambda^3 + 4\lambda^2 - 5\lambda + 4$
- $-\lambda^3 - 3\lambda^2 - 7\lambda - 5$
- $-\lambda^3 + 4\lambda^2 - 5\lambda$
- $-\lambda^3 - 2\lambda^2 - 6\lambda + 2$
- $-\lambda^3 + 2\lambda^2 - 6\lambda + 2$
- None of the above

14 ♣ Suppose  $Q^T A Q = D$  where  $Q^T Q = I$  and  $D$  is diagonal. Which of the following must be true? Mark all that apply.

- $A$  has an eigenvalue that is an imaginary number
- $A^T = A$
- $A$  has all distinct eigenvalues
- There is an orthonormal basis of  $\mathbb{R}^n$  consisting of eigenvectors of  $A$
- The columns of  $Q$  are an orthonormal set
- The eigenvalues of  $A$  are the eigenvalues of  $D$





**Part II: Short Answer Questions:** Write your answers to the questions below in the space provided.

15

0 1 2 3 4 5 6 7 *Administrative Use Only*

- a) Let  $W$  be the subspace of  $\mathbb{R}^5$  spanned by the vectors  $\mathbf{v}_1 = [3 \ 2 \ -1 \ 3 \ -3]^T$  and  $\mathbf{v}_2 = [3 \ 2 \ 2 \ -3 \ 3]^T$ . Find a basis for  $W^\perp$ .

$$\left\{ \begin{array}{l} \\ \\ \\ \\ \end{array} \right\}$$

- b) Let  $W$  be the subspace of  $\mathbb{R}^5$  spanned by the vectors  $\mathbf{v}_1 = [1 \ 1 \ 1 \ 1]^T$  and  $\mathbf{v}_2 = [1 \ 0 \ 1 \ 0]^T$ , and let  $\mathbf{y} = [-1 \ 2 \ 1 \ 0]^T$ . Find the projection of  $\mathbf{y}$  onto  $W$  and the perpendicular component of  $\mathbf{y}$  to  $W$ .

$$\text{proj}_W \mathbf{y} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \quad \text{and} \quad \text{perp}_W \mathbf{y} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

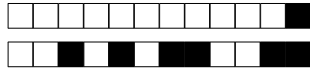
- c) State the precise definition of the following: The set  $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$  is an orthonormal set if

- d) Let

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

What is  $A^{10}$ ?

$$A^{10} = \begin{bmatrix} & \\ & \end{bmatrix}.$$



16

0 1 2 3 4 5 6 7 *Administrative Use Only*

a) Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

Find the inverse of  $A$ .

$$A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}.$$

b) Find a basis for the row space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

$$\left\{ \begin{array}{l} \\ \\ \end{array} \right\}$$

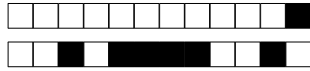
c) Find a basis for the null space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

$$\left\{ \begin{array}{l} \\ \\ \end{array} \right\}$$

d) Give the precise definition of a subspace  $S$  of  $\mathbb{R}^n$ :

e) True or false: every square matrix is the product of elementary matrices?



17

0  1  2  3  4  5  6  7 *Administrative Use Only*

a) If  $\det A = 3$  and  $\det B = -4$ , then  $\det(A^{-1}B^T) =$ \_\_\_\_\_.

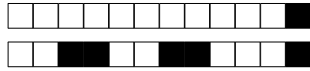
b)  $\det \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} =$ \_\_\_\_\_.

c) If  $A$  has eigenvalues  $\lambda_1, \lambda_2$  with eigenvectors  $\mathbf{u}, \mathbf{v}$  respectively, then

$$A^k(c_1\mathbf{u} + c_2\mathbf{v}) = \text{_____}.$$

d) Write the Spectral Decomposition of the matrix  $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ :

$$A = \text{_____}$$



**Part III: Free Response Questions:** *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

18

0 1 2 3 4 5 6 7 *Administrative Use Only*

Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -4 & -12 \\ 1 & -1 & -3 \\ -3 & 7 & 22 \end{bmatrix}.$$



+1/13/48+

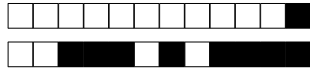
19



Let

$$B = \begin{bmatrix} -2 & -2 & -2 & 1 & -3 \\ 2 & 2 & 1 & 0 & 4 \\ -6 & -6 & -3 & 1 & -9 \\ 12 & 12 & 6 & -2 & 18 \end{bmatrix}.$$

Find bases for each of the following subspaces:  $\text{Row}(B)$ ,  $\text{Col}(B)$ , and  $\text{Null}(B)$ . Clearly label which subspace each basis belongs to.



20



Orthogonally diagonalize the following matrix

$$A = \begin{bmatrix} -5 & -2 & -1 \\ -2 & -2 & 2 \\ -1 & 2 & -5 \end{bmatrix}$$

(i.e. find a diagonal matrix  $D$  and an orthogonal matrix  $Q$  such that  $A = QDQ^T$ ).

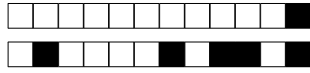


21

0 1 2 3 4 5 6 7 *Administrative Use Only*

Find a QR-decomposition of the matrix

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}.$$



+1/16/45+

22

0 1 2 3 4 5 6 7 *Administrative Use Only*

Find a singular value decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}.$$