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NS 10000001 001

Math 213
Practice Exam I
February 2020

Name: KEY

Section:

Instructor:

Encode your BYU ID in the grid below.

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Instructions

A) Do not write on the barcode area at the top of each page, or near the four circles on each page.

B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.

C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.

D) Multiple choice questions are worth 3 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.

E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.

F) No books, notes, or calculators are allowed.

G) Please do not talk about the test with other students until after the last day of the exam.

CORRECTED

Part I: Multiple Choice Questions: *(3 points each) Questions marked with a ♣ may have more than one correct answer. Mark* **all** *correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but DO NOT mark in the other boxes.*

1 Solve $3\boldsymbol{x} + 2\boldsymbol{u} = \boldsymbol{w}$ for $\boldsymbol{x}$ if $\boldsymbol{u} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$.

☐ $\boldsymbol{x} = \begin{bmatrix} 2/3 \\ -7/3 \\ 2 \end{bmatrix}$.

☐ $\boldsymbol{x} = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$.

☐ $\boldsymbol{x} = \begin{bmatrix} 4/3 \\ 2/3 \\ -1/3 \end{bmatrix}$.

■ $\boldsymbol{x} = \begin{bmatrix} 2/3 \\ 2 \\ -7/3 \end{bmatrix}$.

☐ $\boldsymbol{x} = \begin{bmatrix} 2 \\ 2 \\ -7 \end{bmatrix}$.

2 ♣ Which of the following statements is false? Mark all that apply.

☐ For vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ in $\mathbb{R}^n$ if $\boldsymbol{u} + \boldsymbol{w} = \boldsymbol{v} + \boldsymbol{w}$ then $\boldsymbol{u} = \boldsymbol{v}$.

■ In $\mathbb{R}^3$ if two lines are *not* parallel then they must intersect in a point.

☐ In $\mathbb{R}^3$ if two planes are *not* parallel then they must intersect in a line.

■ For vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ in $\mathbb{R}^n$ if $\boldsymbol{u} \cdot \boldsymbol{w} = \boldsymbol{v} \cdot \boldsymbol{w}$ then $\boldsymbol{u} = \boldsymbol{v}$.

3 ♣ Which if the following matrices is in row echelon form (REF).

■ $\begin{bmatrix} 5 & 2 & -3 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

□ $\begin{bmatrix} 7 & 0 & 6 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

□ $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 3 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

4 Which of the following statements is *false*?

■ $\|\boldsymbol{u} - \boldsymbol{v}\| \leq \|\boldsymbol{u}\| - \|\boldsymbol{v}\|$ for all vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

□ If $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal then $\|\boldsymbol{u} + \boldsymbol{v}\|^2 = \|\boldsymbol{u}\|^2 + \|\boldsymbol{v}\|^2$.

□ $\|\boldsymbol{u} + \boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$ for all vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

□ $\|c\boldsymbol{u}\| = |c|\|\boldsymbol{u}\|$ for all vectors $\boldsymbol{u}$ and scalars c.

□ $|\boldsymbol{u} \cdot \boldsymbol{v}| \leq \|\boldsymbol{u}\|\|\boldsymbol{v}\|$ for all vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

5 ♣ Given that a system of equations has augmented matrix $[A|\boldsymbol{b}]$ that row reduces to the row echelon form $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. Which of the following must be true about the system. Mark all that apply.

- ☐ The columns of A are linearly independent.
- ☐ The columns of A span all of $\mathbb{R}^3$
- ■ The rank of A is 2.
- ☐ The system has 3 free variable.
- ☐ The system is inconsistent.
- ☐ The system has a unique solution.
- ■ The system has infinitely many solutions.

6 Express the vector $\boldsymbol{b} = \begin{bmatrix} 5 \\ -7 \\ 8 \end{bmatrix}$ as a linear combination

$$\begin{bmatrix} 5 \\ -7 \\ 8 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

- ☐ $c_1 = -2, c_2 = 1$
- ■ $c_1 = -2, c_2 = 3$
- ☐ $c_1 = 3, c_2 = -2$
- ☐ $c_1 = 2, c_2 = -3$
- ☐ $c_1 = 3, c_2 = 2$
- ☐ $c_1 = -1, c_2 = 2$
- ☐ $\boldsymbol{b}$ is not a linear combination of these vectors

7 Find the projection of $\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto $\boldsymbol{u} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$.

☐ $\begin{bmatrix} -1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$.

☐ $\begin{bmatrix} -1/3 \\ -1/3 \\ -1/3 \end{bmatrix}$.

☐ $\begin{bmatrix} 0 \\ -1/5 \\ -2/5 \end{bmatrix}$.

☐ $\begin{bmatrix} 0 \\ 0 \\ 2/5 \end{bmatrix}$.

■ $\begin{bmatrix} 0 \\ -1/5 \\ 2/5 \end{bmatrix}$.

☐ $\begin{bmatrix} 1/3 \\ -1/3 \\ -1/3 \end{bmatrix}$.

☐ $\begin{bmatrix} -1/3 \\ 0 \\ -1/3 \end{bmatrix}$.

☐ $\begin{bmatrix} -1/5 \\ 0 \\ 2/5 \end{bmatrix}$.

8 Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & s \\ 2 & 9 & 5 \end{bmatrix}.$$

What value(s) of s will make the matrix A noninvertible? Select one answer only.

- ■ $s = 1$
- ☐ $s = 2$
- ☐ $s = 0$
- ☐ Any value except for $s = 1$.
- ☐ Any value except for $s = 0$.
- ☐ $s = 5$
- ☐ Any value except for $s = 2$.
- ☐ Any value except for $s = -1$.

9 ♣ Which of the following matrices are invertible? Mark all that apply.

- ☐ $\begin{bmatrix} 1 & -3 \\ 2 & -6 \end{bmatrix}$
- ■ $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & 2 & 5 \end{bmatrix}$
- ■ $\begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$
- ■ $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 0 & -4 & 0 \\ 2 & 0 & 8 \\ 0 & 7 & 0 \end{bmatrix}$

10 ♣ Which of the following sets of vectors spans all of $\mathbb{R}^n$ for the appropriate n?

■ $\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}, \begin{bmatrix}5\\6\end{bmatrix} \right\}$

■ $\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\4\\-2\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix}1\\2\end{bmatrix} \right\}$

■ $\left\{ \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\4\\-2\end{bmatrix}, \begin{bmatrix}0\\0\\7\end{bmatrix} \right\}$

11 ♣ Let $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$ be vectors in $\mathbb{R}^n$, and let A be the matrix whose columns are $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$. Which of the following is always true? Mark all that apply

■ If $rank(A) = k$, then $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$ are linearly independent.

■ If $\boldsymbol{v}_1 = 2\boldsymbol{v}_2$ then $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ is a linearly dependent set.

□ If $k < n$, then $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$ are linearly independent.

□ If $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$ are linearly independent, they must span all of $\mathbb{R}^n$.

■ If $k > n$, then $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$ are linearly dependent.

□ If $A\boldsymbol{x} = \boldsymbol{0}$ is consistent, then $\boldsymbol{v}_1, ..., \boldsymbol{v}_k$ are linearly independent.

12 Find the inverse of the elementary matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

- [ ] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- [x] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10 **DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

a) True or (False) A homogeneous system always has infinitely many solutions.

b) (True) or False: The vectors $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}$ span all of $\mathbb{R}^2$.

c) If $B = \begin{bmatrix}1 & -4\\2 & 2\end{bmatrix}$, then $\det B =$ $2-(-8)=10$

d) Given that $\boldsymbol{x} = \begin{bmatrix}3\\1\\-3\end{bmatrix}$, $\boldsymbol{y} = \begin{bmatrix}3\\-5\\1\end{bmatrix}$, $\boldsymbol{z} = \begin{bmatrix}3\\2\\1\end{bmatrix}$, calculate

$$\frac{(\boldsymbol{x}+\boldsymbol{y})\cdot\boldsymbol{z}}{\boldsymbol{x}\cdot(\boldsymbol{y}+\boldsymbol{z})} = \frac{8}{9}$$

e) Find the equation for the plane passing through the point $P = (1, 0, 2)$ and with normal vector $\begin{bmatrix}-1\\0\\1\end{bmatrix}$.

$$\begin{bmatrix}-1\\0\\1\end{bmatrix}\cdot\left(\begin{bmatrix}x\\y\\z\end{bmatrix}-\begin{bmatrix}1\\0\\2\end{bmatrix}\right)=0$$

$$-(x-1)+(z-2)=0$$

$$-x+z=1$$

14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10 DON'T MARK

Fill in the blank with the appropriate answer. 2 points per answer.

a) True or false: If A, B and C are 3×3 matrices and $AB = AC$, then $B = C$. ________

b) True or false: The matrices $\begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ commute. ________

c) True or false: If A is a 3×4 matrix, then AA^T is symmetric. ________

d) How many solutions are there to a system of linear equations with augmented matrix that row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & 3 & -4 \end{array}\right]$? Infinitely Many

e) Write the system

$$\begin{aligned} 2x - 3y &= -1 \\ -5x + 2y &= 3 \end{aligned}$$

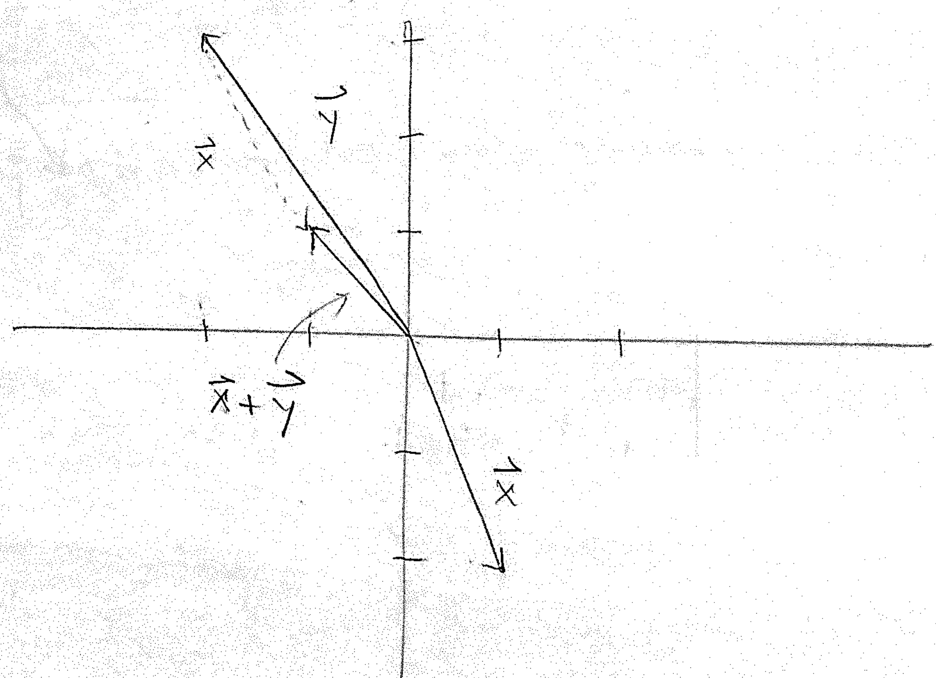
as a vector equation (a linear combination of vectors equal to some vector).

$$x \begin{bmatrix} 2 \\ -5 \end{bmatrix} + y \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

15 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 DON'T MARK

Geometrically show what is meant by adding the two vectors $\boldsymbol{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\boldsymbol{y} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.



16 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 DON'T MARK

Solve the system of equations. If the system is inconsistent, state so. If there are infinitely many solutions, express them in parametric-vector form.

$$x - 2y + 3z = -1$$
$$-2x + 3y + z = 0$$
$$-x + y + 4z = -1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ -2 & 3 & 1 & 0 \\ -1 & 1 & 4 & -1 \end{array}\right] \begin{array}{l} \\ R_2+2R_1 \\ R_3+R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & -1 & 7 & -2 \\ 0 & -1 & 7 & -2 \end{array}\right] \begin{array}{l} \\ -R_2 \\ R_3-R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 1 & -7 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1+2R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & -11 & 3 \\ 0 & 1 & -7 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{RREF}$$

$x - 11z = 3$
$y - 7z = 2$
$z = t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3+11t \\ 2+7t \\ t \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 11 \\ 7 \\ 1 \end{bmatrix}$$

17 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 DON'T MARK

Find the inverse of the matrix

$$A = \begin{bmatrix} -2 & -1 & 0 & 0 \\ -3 & -1 & 1 & 1 \\ -6 & -2 & 1 & 1 \\ 6 & 2 & -1 & 0 \end{bmatrix}.$$

$$\left[\begin{array}{cccc|cccc} -2 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -3 & -1 & 1 & 1 & 0 & 1 & 0 & 0 \\ -6 & -2 & 1 & 1 & 0 & 0 & 1 & 0 \\ 6 & 2 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 2R_2 \\ R_4 + R_3 \end{array}$$

$$\left[\begin{array}{cccc|cccc} -2 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ -R_2 \\ R_3 + R \\ \end{array}$$

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -2 & -2 & 3 & -2 & 0 & 0 \\ 1 & 0 & -1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} -R_1 \\ R_1 \Leftrightarrow R_2 \\ -R_3 \\ \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 + R_4 \\ R_2 - 2R_4 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 0 & 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 0 & -3 & 2 & -2 & -2 \\ 0 & 0 & 1 & 0 & 0 & 2 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 + R_3 \\ R_2 - 2R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -3 & -2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & -1 & 0 \\ -3 & -2 & 2 & 0 \\ 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 DON'T MARK

Consider the matrices

$$A_1 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \text{and} \quad A_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Is the matrix $B = \begin{bmatrix} 0 & 4 \\ 7 & 9 \end{bmatrix}$ in Span $\{A_1, A_2, A_3\}$? Justify your answer.

$$x_1 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + x_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 7 & 9 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 4 \\ 3 & 1 & 0 & 7 \\ 4 & 1 & 1 & 9 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-3R_1 \\ R_4-4R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & 4 \\ 0 & -2 & -3 & 7 \\ 0 & -3 & -3 & 9 \end{array}\right] \begin{array}{l} \\ -R_2 \\ R_2-2R_2 \\ R_3-3R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & -3 \end{array}\right] \begin{array}{l} \\ \\ \\ R_4-3R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system is consistent, therefore B is in Span(A_1, A_2, A_3)