CORRECTED

NS 40121930 001

Math 213
Exam I
February 12–14, 2020

Name: KEY

Section:

Instructor:

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Instructions

A) Do not write on the barcode area at the top of each page, or near the four circles on each page.

B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.

C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.

D) Multiple choice questions are worth 4 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.

E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.

F) No books, notes, or calculators are allowed.

G) Please do not talk about the test with other students until after the last day of the exam.

CORRECTED

Part I: Multiple Choice Questions: *(4 points each) Questions marked with a ♣ may have more than one correct answer. Mark* ***all*** *correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but DO NOT mark in the other boxes.*

1 ♣ For vectors $\boldsymbol{u}, \boldsymbol{v}$, and $\boldsymbol{w}$ in $\mathbb{R}^n$ ($n \geq 2$), and c a real scalar, which of the following are valid, well-defined expressions? Mark all that apply?

- ☐ $\boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{w}$.
- ☐ $\boldsymbol{u} \cdot (\boldsymbol{v} \cdot \boldsymbol{w})$.
- ☐ $\|\boldsymbol{u} \cdot \boldsymbol{v}\|$.
- ■ $(c\boldsymbol{u} - \boldsymbol{w}) \cdot \boldsymbol{v}$.
- ■ $c(\boldsymbol{u} + \boldsymbol{w})$.

2 If $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal vectors then what can we say about the projection of $\boldsymbol{u}$ onto $\boldsymbol{v}$?

- ☐ The projection will not be orthogonal to $\boldsymbol{v}$.
- ■ The projection is the zero vector.
- ☐ The projection will be longer than $\boldsymbol{v}$.
- ☐ The projection will not be orthogonal to $\boldsymbol{u}$.
- ☐ The projection will be longer than $\boldsymbol{u}$.
- ☐ We can't conclude anything from the information given.

3 Which of the following are orthogonal to the vector $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

■ $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

□ $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

□ $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

□ $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

□ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

□ $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

4 ♣ Which if the following matrices is in reduced row echelon form (RREF). Mark all that apply.

□ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

□ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

□ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

□ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

5 ♣ Given that a system of equations has augmented matrix $[A|\boldsymbol{b}]$ that row reduces to the row echelon form $\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 2 & 3 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$. Which of the following must be true about the system. Mark all that apply.

■ The rank of A is 3.

□ The system is inconsistent.

■ The columns of A span all of $\mathbb{R}^3$

□ The system has a unique solution.

■ The system has infinitely many solutions.

□ The columns of A are linearly independent.

■ The system has 1 free variable.

6 ♣ Which of the following sets of vectors is linearly independent? Mark all that apply.

- ■ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$
- Wrong → ■ $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$
- ☐ $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \right\}$
- ■ $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

7 Express the vector $\boldsymbol{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

- ☐ $c_1 = 2, c_2 = -3$
- ☐ $c_1 = -1, c_2 = 2$
- ☐ $c_1 = 3, c_2 = -2$
- ☐ $c_1 = 3, c_2 = 2$
- ☐ $\boldsymbol{b}$ is not a linear combination of these vectors
- ■ $c_1 = 2, c_2 = -1$
- ☐ $c_1 = -2, c_2 = 1$

8 ♣ Let $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^3$, and let A be the 3×2 matrix with $\boldsymbol{u}$ and $\boldsymbol{v}$ as its columns. Which of the following must be true? Mark all that apply.

- [x] If $\boldsymbol{b} \in span(\boldsymbol{u}, \boldsymbol{v})$, then $A\boldsymbol{x} = \boldsymbol{b}$ has a solution.
- [ ] $span(\boldsymbol{u}, \boldsymbol{v})$ is a plane.
- [x] $span(\boldsymbol{u}, \boldsymbol{v})$ is NOT all of $\mathbb{R}^3$.
- [x] If $\boldsymbol{b} \in span(\boldsymbol{u}, \boldsymbol{v})$, then $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{b}\}$ is linearly dependent.
- [x] If at least one of $\boldsymbol{u}$ and $\boldsymbol{v}$ is not the zero vector, then $span(\boldsymbol{u}, \boldsymbol{v})$ contains infinitely many vectors.
- [ ] If $rank(A) = 2$, then $\boldsymbol{u} = c\boldsymbol{v}$ for some constant c.

9 ♣ Let A be an $n \times n$ matrix. Which of the following facts are equivalent to A being invertible? Mark all that apply.

- [ ] A has at least one positive entry.
- [x] There is an $n \times n$ matrix C such that AC is the $n \times n$ identity matrix.
- [x] The columns of A span $\mathbb{R}^n$.
- [x] rank $A = n$
- [ ] A commutes with B for all $n \times n$ matrices B.
- [x] A has a pivot in every row.

10 ♣ Which of the following matrices are invertible? Mark all that apply.

☐ $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 4 & 4 \end{bmatrix}$

■ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

11 ♣ Which of the following are elementary matrices? Mark all that apply.

■ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 15 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

■ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12 ♣ Let A, B, and C be invertible $n \times n$ matrices. Which of the following must be true? Mark all that apply.

- ■ $A^T B^T$ is invertible.
- ■ $(A^{-1})^T = (A^T)^{-1}$
- ■ $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
- □ $((AB)^{-1})^T = (B^{-1})^T(A^{-1})^T$
- ■ If $AB = AC$ then $B = C$.
- □ $(ABC)^T = A^T B^T C^T$
- ■ $(AB)^{-1} = B^{-1}A^{-1}$

Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10 *Administrative Use Only*

a) True of False: A homogeneous system of equations must be consistent. True

b) True or False: The vectors $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}$ span all of $\mathbb{R}^3$. False

c) How many solutions are there to a system of linear equations with augmented matrix that row reduces to $\left[\begin{array}{cc|c}1 & 0 & 2\\0 & 2 & -4\\0 & 0 & 3\end{array}\right]$? None

d) Write the following system of equation

$$\begin{array}{rcrcrcr}3x & - & y & + & 2z & = & 7\\x & + & 2y & - & z & = & 13\\2x & + & y & + & z & = & 1\\ & & y & + & z & = & -3\end{array}$$

as a matrix-vector equation $A\boldsymbol{x} = \boldsymbol{b}$ (not as an augmented matrix):

$$\begin{bmatrix}3 & -1 & 2\\1 & 2 & -1\\2 & 1 & 1\\0 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}7\\13\\1\\-3\end{bmatrix}$$

e) If $A = \begin{bmatrix}3 & 1\\-2 & 1\end{bmatrix}$, then $A^{-1} = \begin{bmatrix}1/5 & -1/5\\2/5 & 3/5\end{bmatrix}$.

14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ■12 *Administrative Use*

Fill in the blank with the appropriate answer. 2 points per answer unless stated otherwise.

a) Let

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}.$$

Compute the following matrix products (if the product is not defined write *not defined* in the space provided).

$$AB = \begin{bmatrix} 2 & 8 & -7 \\ 1 & 3 & -1 \end{bmatrix}, \quad AC = \begin{bmatrix} \text{Not Defined} \end{bmatrix}, \quad CA = \begin{bmatrix} 7 & 11 & 1 \\ 3 & -1 & 3 \end{bmatrix}.$$

b) Given that $\boldsymbol{x} = \boldsymbol{u} + 3\boldsymbol{v}$ where $\boldsymbol{u} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$, find the value of $\boldsymbol{x} \cdot \boldsymbol{u}$.

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c) Find the equation for the line passing through the point $P = (0, 1)$ and with normal vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x - 0 \\ y - 1 \end{pmatrix}$ $\quad x + 2(y-1) = 0$

$y = -\frac{1}{2}x + 1$

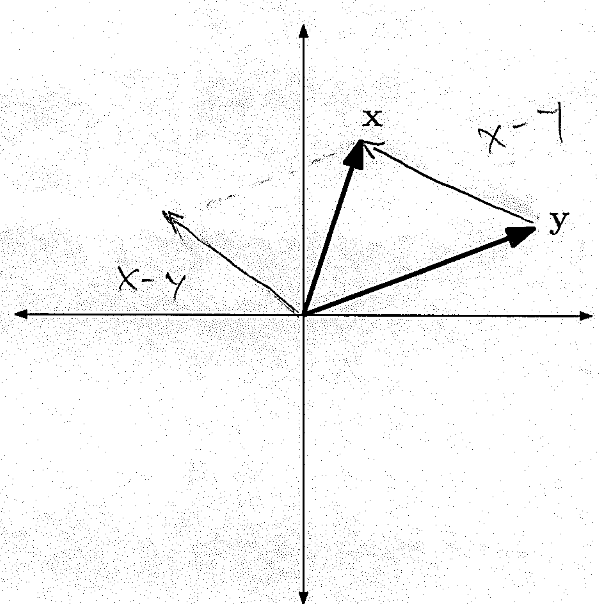
d) Find the projection of $\boldsymbol{v}$ onto $\boldsymbol{u}$ where $\boldsymbol{u} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$.

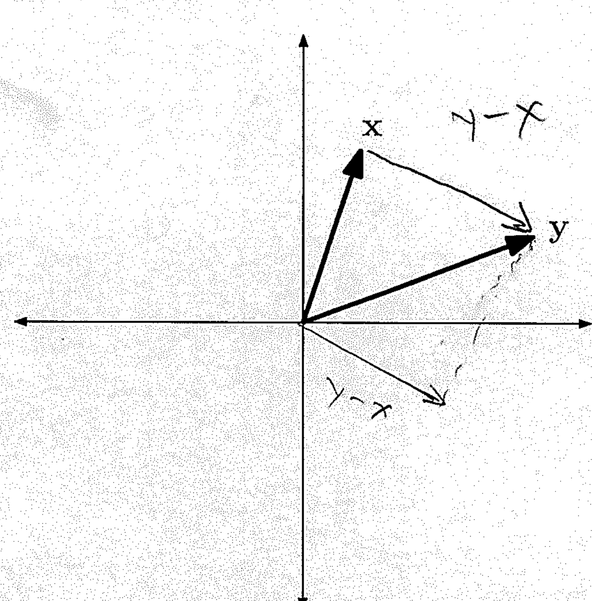
$$\frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{\begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}}{\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \frac{-4}{5} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}.$$

Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

15 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ■6 *Administrative Use Only*

Geometrically show what is meant by subtracting the two vectors **x** and **y** from each other, by plotting **x** − **y** on the top axes and **y** − **x** on the bottom axes.





 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 *Administrative Use Only*

Solve the system of equations. If it is inconsistent, state so. If there are infinitely many solutions, express them in parametric-vector form.

$$x - 2y - z = 1$$
$$2x + y + 3z = 12$$
$$3x - 4y - z = 7$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 2 & 1 & 3 & 12 \\ 3 & -4 & -1 & 7 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & 5 & 10 \\ 0 & 2 & 2 & 4 \end{array}\right] \begin{array}{l} \\ \frac{1}{5}R_2 \\ R_3 - \frac{2}{5}R_2 \end{array} \quad \overset{R_1 + 2R_2}{\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Infinitely many solutions
z is free

$$x + z = 5$$
$$y + z = 2$$
$$z = t$$

$$x = 5 - t$$
$$y = 2 - t$$
$$z = t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

17 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 *Administrative Use Only*

Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 1 & 6 \\ -1 & 0 & 3 \end{bmatrix}.$$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 4 & 1 & 0 & 0 \\ -1 & 1 & 6 & 0 & 1 & 0 \\ -1 & 0 & 3 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} R_1 \leftrightarrow R_2 \\ -R_2 \\ R_3 - R_2 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -6 & 0 & -1 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 \\ 0 & -1 & -3 & 0 & -1 & 1 \end{array}\right) \begin{array}{l} R_1 + R_2 \\ \\ R_3 + R_2 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -1 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right) \begin{array}{l} R_1 + 2R_3 \\ R_2 - 4R_3 \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 2 \\ 0 & 1 & 0 & -3 & 4 & -4 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 3 & -3 & 2 \\ -3 & 4 & -4 \\ 1 & -1 & 1 \end{pmatrix}$$

 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 *Administrative Use Only*

Consider the matrices

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \text{and} \quad A_3 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Is the set $\{A_1, A_2, A_3\}$ linearly independent? Justify your answer.

$$x_1 \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right) \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right) \begin{array}{l} R_2 + R_4 \\ R_3 \leftrightarrow R_4 \\ \frac{1}{2}R_4 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) R_2 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \text{REF}$$

Each column has a pivot, so there is only the trivial solution, thus $\{A_1, A_2, A_3\}$ is a linearly independent set.