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Math 213
Exam I
October 2019

Name: KEY

Section:

Instructor: Department Exam

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Instructions

A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
D) Multiple choice questions are worth 4 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
F) No books, notes, or calculators are allowed.
G) Do not talk about the test with others until after the last day of the exam.
H) The last two pages of the exam may be used as scratch paper.

Part I: Multiple Choice Questions: *(4 points each) Questions marked with a ♣ may have more than one correct answer. Mark **all** correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but DO NOT mark in the other boxes.*

1 Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 4 & 5 & -3 \\ -2 & -1 & -4 & -5 & 4 \\ -5 & -3 & -11 & -14 & 9 \end{bmatrix}.$$

Which of the following matrices is the reduced row echelon form of A?

☐ $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2 ♣ Which of the augmented matrices will correspond to systems of linear equations with more than one solution? Mark all that apply.

□ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

■ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 \end{array}\right]$

□ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$

■ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right]$

□ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -3 \end{array}\right]$

■ $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 1 & 1 & -3 & 0 \end{array}\right]$

3 ♣ Which of the following sets of vectors are linearly independent? Mark all that apply.

■ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

□ $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

■ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

4 ♣ Suppose that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{b} \in \mathbb{R}^5$, and that the system of equations corresponding to the augmented matrix

$$\left[\begin{array}{ccc|c} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{b} \end{array} \right]$$

is consistent. Which of the following *must* be true? Mark all that apply.

- ■ There are scalars c_1, c_2, c_3 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{b}$.
- ☐ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \mathbb{R}^5$.
- ■ $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{b}\}$ is linearly dependent.
- ☐ $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.
- ☐ The rank of $\left[\begin{array}{ccc} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{array} \right]$ is equal to 3.
- ■ $\mathbf{b} \in$ Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.
- ☐ The solution to $\left[\begin{array}{ccc|c} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{b} \end{array} \right]$ is unique.

5 The points

$$A = (1, 2, 3), B = (2, 3, 5), C = (6, 4, -1), D = (1, 2, -1) \in \mathbb{R}^3$$

form the corners of a four-sided figure in $\mathbb{R}^3$ whose sides give the vectors $\vec{AB}, \vec{BC}, \vec{CD}, \vec{DA}$. At which of the points A, B, C, D are the vectors which meet at the corner orthogonal?

- ■ D
- ☐ C
- ☐ B
- ☐ A
- ☐ none of them

6 Let $\mathbf{v}_1 = [1, 2, 3], \mathbf{v}_2 = [2, 3, 5], \mathbf{v}_3 = [6, 4, -1], \mathbf{v}_4 = [1, 2, -1] \in \mathbb{R}^3$ be vectors. Let $\mathbf{v} = [x, y, z]$. Solve the vector equation for $\mathbf{v}$:

$$2\mathbf{v}_1 - 4\mathbf{v}_2 + \mathbf{v} = 7\mathbf{v}_3 - \mathbf{v}_4.$$

- ☐ $[-47, -34, -8]$
- ☐ $[-47, -34, 8]$
- ■ $[47, 34, 8]$
- ☐ $[47, -34, 8]$
- ☐ $[47, 34, -8]$
- ☐ $[-47, 34, -8]$

7 ♣ If $\mathbf{v}_1 = [1, 1, 3], \mathbf{v}_2 = [2, 0, 6]$, which of the following are linear combinations of $\mathbf{v}_1, \mathbf{v}_2$?

- ☐ $[1, -1, 5]$
- ■ $[-3, -3, -9]$
- ☐ $[3, 1, 8]$
- ■ $[1, 3, 3]$
- ■ $[2, 0, 6]$
- ■ $[0, 0, 0]$

8
Let $\mathbf{v} = [0, -1], \mathbf{w} = [\sqrt{3}, -1]$ and find the angle between $\mathbf{v}$ and $\mathbf{w}$.

- ☐ $2\pi/3$
- ☐ $\pi/4$
- ☐ $\pi/2$
- ☐ 0
- ■ $\pi/3$
- ☐ $\pi/6$

9 Let A, B, and C be $n \times n$ matrices and r a scalar. Which of the following properties of matrix operations does *not* always hold?

- ☐ $r(A+B) = rA + rB$
- ☐ $A + B = B + A$
- ■ $AB = BA$
- ☐ $(rA)B = A(rB)$
- ☐ $(AB)C = A(BC)$
- ☐ $A(B+C) = AB + AC$

10 ♣ Which of the following statements is always true? Mark all that apply.

- ☐ If the columns of an $m \times n$ matrix span $\mathbb{R}^m$ then the columns of A are linearly independent.
- ■ If the matrix B^T is invertible then B is invertible.
- ■ If A is $n \times n$ and invertible, then $\text{rank}(A) = n$.
- ☐ If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$ in $\mathbb{R}^n$ then $A\mathbf{x} = \mathbf{0}$ has a non-trivial solution.
- ☐ If A is an $m \times n$ matrix with $m > n$ then A is invertible.
- ■ If A is an $n \times n$ matrix and $AB = I_n$, then $BA = I_n$.

11 ♣ Let A be an $m \times n$ matrix. Which of the following must be true? (Mark all that apply.)

- ■ A^T is $n \times m$.
- ☐ $A + A^T$ is defined and symmetric.
- ■ $(A^T)^T = A$.
- ☐ None of these is correct.
- ■ AA^T is symmetric.
- ☐ A^T is symmetric.

12 Find the inverse of the following elementary matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

- ☐ $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
- ■ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

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Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ■12 *Administrative Use*

Fill in the blank with the appropriate answer. 2 points per answer.

a) Find the projection of the point $(1, 2)$ onto the line with equation $y = -x$.

$\frac{1}{2}[-1, 1]$

b) Consider the lines

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

These lines intersect at the point $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

c) The reduced row echelon form of the matrix $\begin{bmatrix} 1 & 0 & 3 & -2 \\ -3 & 1 & -8 & 6 \\ 1 & -1 & 2 & -2 \end{bmatrix}$ is

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

d) Define what it means for the set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ to be linearly independent:

If $c_1\vec{v}_1 + \cdots + c_p\vec{v}_p = \vec{0}$ then $c_1 = 0, \ldots, c_p = 0$.

e) If A and B are invertible, then $(AB)^{-1} =$ $B^{-1}A^{-1}$.

f) True or False: If A is a 2×2 matrix and $A^2 = 0$ then $A = 0$. False

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14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ■12 *Administrative Use*

Fill in the blank with the appropriate answer. 2 points per answer unless stated otherwise.

a) Let $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$. If $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix}$, find the value of k so that $\mathbf{b} \in \text{Span}\{\mathbf{v}, \mathbf{u}\}$.

$k =$ 5

b) (4 points) If A is a 5×4 matrix and B is a 4×3 matrix and C a 3×6 matrix, for each of the following products, state the size (dimensions) of the product, or if the product is not defined, state that it is undefined.

AB: 5×3 BA: undefined AC: undefined BC: 4×6

c) Find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. $A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$.

d) Write the following system as a matrix equation $A\mathbf{x} = \mathbf{b}$

$$3x_1 - 3x_2 - x_3 = 0, \; 2x_2 - 4x_3 + 5x_4 + 6 = 0.$$

$$\begin{bmatrix} 3 & -3 & -1 & 0 \\ 0 & 2 & -4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -6 \end{bmatrix}$$

e) Compute the matrix product

$$\begin{bmatrix} -3 & -4 & 6 \\ 0 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 15 & 4 \\ 3 & 13 \end{bmatrix}.$$

Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

15 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 *Administrative Use Only*

If $\mathbf{u} \in \mathbb{R}^n$ is orthogonal to $\mathbf{v} \in \mathbb{R}^n$ and $\mathbf{w} \in \mathbb{R}^n$, show that $\mathbf{u}$ is orthogonal to $2\mathbf{v} - 8\mathbf{w}$.

Since $\vec{u}$ is orthogonal to $\vec{v}$ and $\vec{w}$,

$\vec{u} \cdot \vec{v} = 0$ and $\vec{u} \cdot \vec{w} = 0$.

Thus, by properties of dot products

$$\vec{u} \cdot (2\vec{v} - 8\vec{w}) = \vec{u} \cdot (2\vec{v}) - \vec{u} \cdot (8\vec{w})$$
$$= 2(\vec{u} \cdot v) - 8(\vec{u} \cdot \vec{w})$$
$$= 2(0) - 8(0) = 0$$

Thus $\vec{u} \cdot (2\vec{v} - 8\vec{w}) = 0$

so $\vec{u}$ is orthogonal to $2\vec{v} - 8\vec{w}$

16 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 *Administrative Use Only*

Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ be non-zero vectors that are orthogonal. Explain why

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \mathbf{0}.$$

(*Hint: this should be a precise explanation using the formula for a projection.*)

$$\text{proj}_{\vec{u}}(\vec{v}) = \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u}$$

Since $\vec{u}$ and $\vec{v}$ are orthogonal,

$\vec{u} \cdot \vec{v} = 0$

Thus $\text{proj}_{\vec{u}}(\vec{v}) = \frac{0}{\vec{u} \cdot \vec{u}} \vec{u} = \vec{0}$.

17 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 *Administrative Use Only*

Consider the planes in $\mathbb{R}^3$ defined by the equations

$$-x - 2z = 3 \qquad \text{and} \qquad 2x + y + 7z = -7.$$

Find the *vector form* equation of the line of intersection between these two planes.

$$\left[\begin{array}{ccc|c} -1 & 0 & -2 & 3 \\ 2 & 1 & 7 & -7 \end{array}\right] \begin{array}{l} 2R_1 + R_2 \\ -R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$x \quad + 2z = -3$ $\qquad z$ free $\quad z = t$

$y + 3z = -1$

$x = -3 - 2t$

$y = -1 - 3t$

$z = t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$$

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 *Administrative Use Only*

Find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] -R_3+R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right] -R_2+R_3$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right] -R_1+R_2$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

Therefore

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$