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NS 10000001 **001**

Math 213
Practice Exam I
October 2019

Name: KEY
Section:
Instructor:

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Instructions

A) Do not write on the barcode area at the top of each page, or near the four circles on each page.
B) Fill in your name, the correct boxes for your BYU ID and for the correct answer on the multiple choice completely.
C) The multiple choice questions that are marked with a ♣ may have more than one correct answer. You should mark all correct answers. All other questions have only one correct answer.
D) Multiple choice questions are worth 3 points each. For multiple choice questions with more than one correct answer, each option will be graded with equal weight.
E) For questions which require a written answer, show all your work in the space provided and justify your answer. Simplify your answers where possible.
F) No books, notes, or calculators are allowed.
G) Please do not talk about the test with other students until after the last day of the exam.

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Part I: Multiple Choice Questions: *(3 points each) Questions marked with a ♣ may have more than one correct answer. Mark* ***all*** *correct answers. The other questions have one correct answer. Choose the best answer for each multiple choice question. Fill in the box completely for each correct answer, but DO NOT mark in the other boxes.*

1 ♣ Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Which of the following matrices are row equivalent to A? Mark all that apply.

■ $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 2 & 1 & 5 & 0 & 3 \end{bmatrix}$

■ $\begin{bmatrix} 2 & 0 & 4 & 0 & 2 \\ 0 & 2 & 2 & 0 & 2 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

■ $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 1 & 1 & 3 & 0 & 2 \end{bmatrix}$

■ $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 & 1 \end{bmatrix}$

2 ♣ Which of the following augmented matrices will correspond to systems of linear equations with a unique solution? Mark all that apply.

☐ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 4 \end{array}\right]$

■ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & 7 \end{array}\right]$

☐ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 7 \\ 3 & 6 & 9 & 12 \end{array}\right]$

☐ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

☐ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$

■ $\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 1 & 1 & -3 \end{array}\right]$

3 ♣ Which of the following sets of vectors are linearly independent? Mark all that apply.

■ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\}$

■ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$

■ $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

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4 ♣ Which of the following vectors is in

$$\text{Span}\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix}\right\}?$$

Mark all that apply.

- ☐ $\begin{bmatrix}1\\-1\\2\end{bmatrix}$
- ■ $\begin{bmatrix}0\\0\\0\end{bmatrix}$
- ■ $\begin{bmatrix}1\\0\\1\end{bmatrix}$
- ■ $\begin{bmatrix}-3\\2\\-3\end{bmatrix}$
- ☐ $\begin{bmatrix}2\\0\\-1\end{bmatrix}$
- ■ $\begin{bmatrix}5\\5\\5\end{bmatrix}$

5 ♣ Suppose that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \in \mathbb{R}^4$, and that the set of vectors

$$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$$

is linearly independent. Which of the following *must* be true? Mark all that apply.

- ☐ One of the vectors $\mathbf{v}_j$ can be expressed as a linear combination of the other vectors.
- ■ The matrix $\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4\end{bmatrix}$ has a pivot in every column.
- ☐ For any vector $\mathbf{b} \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$, the equation $c_1\mathbf{v}_1+c_2\mathbf{v}_2+c_3\mathbf{v}_3+c_4\mathbf{v}_4 = \mathbf{b}$ has infinitely many solutions.
- ■ The rank of the matrix $\begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4\end{bmatrix}$ is 4.
- ■ The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.
- ■ The system of equations corresponding to the augmented matrix $\left[\begin{array}{cccc|c}\mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 & \mathbf{b}\end{array}\right]$ is consistent for all choices of $\mathbf{b} \in \mathbb{R}^4$.

6 ♣ The points $A = (-1, -3, 5), B = (12, 4, 6), C = (16, 18, 22), D = (1, 2, 0) \in \mathbb{R}^3$ form the corners of a four-sided figure whose sides give the vectors $\vec{AB}, \vec{BC}, \vec{CD}, \vec{DA}$. At which of the points A, B, C, D are these vectors orthogonal?

☐ B

☐ C

☐ none of them

☐ A

■ D

7 Let $v_1 = [1, 2, -3], v_2 = [2, 3, -5], v_3 = [6, -4, -1], v_4 = [1, 2, -1] \in \mathbb{R}^3$ be vectors. Let $v = [x, y, z]$. Solve the vector equation

$$-2v_1 + 4v_2 + v = 7v_3 + 2v_4.$$

☐ [38,32,-5]

☐ [-38,-32,5]

■ [38,-32,5]

☐ [-38,32,5]

☐ [-38,-32,-5]

8 Let $v = [1, 1], w = [1, 0]$ and find the angle between v and w

☐ $\pi/2$

☐ $2\pi/3$

☐ 0

■ $\pi/4$

☐ $\pi/6$

☐ $\pi/3$

9 Let A, B, and C be $n \times n$ matrices and r a scalar. Which of the following properties of matrix transpose does *not* always hold.

■ $(AB)^T = A^T B^T$

☐ $(rA)^T = rA^T$

☐ $(A + B)^T = A^T + B^T$

☐ $(A^T)^{-1} = (A^{-1})^T$ (assuming A is invertible)

☐ $(A^T)^T = A$

10 ♣ Which of the following statements are always true? Mark all that apply.

- ■ If A is $n \times n$ and invertible, then $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b} \in \mathbb{R}^{\mathbf{n}}$.
- ■ If A is $n \times n$ and invertible, then its columns span $\mathbb{R}^n$.
- ☐ If A is $m \times n$ and $n < m$, then the columns of A are linearly independent.
- ■ If A is $m \times n$ and $n > m$, then the columns of A are linearly dependent.
- ☐ If A is $m \times n$ and $n > m$, then the columns of A span $\mathbb{R}^m$.
- ☐ If A is $m \times n$ and $\text{rank}(A) = n$, then A is invertible.

11 ♣ Given that A and B are invertible, which of the following statements are always true? Mark all that apply.

- ■ $(-A)^{-1} = -(A^{-1})$
- ■ $AA^{-1} = A^{-1}A$
- ☐ $(AB)^{-1} = A^{-1}B^{-1}$
- ☐ $(A^{-1})^{-1} = A^{-1}$
- ☐ $(A+B)^{-1} = A^{-1} + B^{-1}$
- ☐ None of these is true.

12 Find the inverse of the following elementary matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

☒ $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

☐ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

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Part II: Short Answer Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ■12 **DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

a) Find the projection of the point $(1, -1)$ onto the line with equation $y = 2x$.

$-\frac{1}{5}[1,2]$

b) The vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} 7 \\ -2 \\ -5 \\ 2 \end{bmatrix}$$

form a linearly dependent set. Write a dependence relation for these vectors:

$3v_1 - 4v_2 - v_3 = 0$.

c) Define the span of a set of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$: span$(\vec{v}_1, \cdots \vec{v}_p) =$

set of all linear combinations $c_1\vec{v}_1 + \cdots + c_p\vec{v}_p$.

d) Circle the pivot positions in the matrix

$$\begin{bmatrix} \enclose{circle}{-1} & 1 & -1 & 1 & 2 \\ 0 & 0 & \enclose{circle}{5} & 0 & 5 \\ 0 & 0 & 0 & \enclose{circle}{1} & 1 \end{bmatrix}.$$

e) If A is $m \times n$ and B is $n \times k$ then AB is m × k.

f) Is the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ invertible? No

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14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 ☐11 ■12 **DON'T MARK**

Fill in the blank with the appropriate answer. 2 points per answer.

a) Let $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 4 \end{bmatrix}$. If $\mathbf{w} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ k \end{bmatrix}$, find the value of k so that $\{\mathbf{v}, \mathbf{u}, \mathbf{w}\}$ forms a linearly dependent set.

$k =$ 8

b) [✓ True] or False: If $A^2 = I$ then A is invertible.

c) True or [✓ False]: If A is $m \times n$ and B is $k \times m$, then BA is undefined.

d) Write the following system as a matrix equation $A\mathbf{x} = \mathbf{b}$:

$$2x + 3y = 7, \quad -3y + 5x = -2$$

$2x+3y=7$

$5x-3y=-2$

$$\begin{bmatrix} 2 & 3 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

e) $(A^n)^{-1} =$ $(A^{-1})^n$

f) Compute the matrix product

$$\begin{bmatrix} 1 & 2 \\ -2 & 3 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & -6 \\ -1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 10 & 4 \\ -7 & 1 & 27 \\ -7 & 11 & 31 \end{bmatrix}$$

Part III: Free Response Questions: *Neatly write complete solutions for these problems directly on the exam paper. Work on scratch paper will not be graded.*

15 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 DON'T MARK

Describe all vectors that are orthogonal to $(-7, 10) \in \mathbb{R}^2$.

$$\begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} -7 \\ 10 \end{bmatrix} = 0$$

$$-7x + 10y = 0$$

$$10y = 7x$$

$$y = \frac{7}{10} x$$

All vectors of the form $\begin{bmatrix} x \\ 7/10\, x \end{bmatrix}$ are orthogonal to $\begin{bmatrix} -7 \\ 10 \end{bmatrix}$

i.e. all multiples of $\begin{bmatrix} 1 \\ 7/10 \end{bmatrix}$

(or all multiples of $\begin{bmatrix} 10 \\ 7 \end{bmatrix}$)

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☒7 **DON'T MARK**

Let $\vec{u}, \vec{v} \in \mathbb{R}^n$ be non-zero vectors. Explain why

$$\text{proj}_{\vec{u}}(\vec{v}) = \text{proj}_{\vec{u}}(\text{proj}_{\vec{u}}(\vec{v})).$$

The formula for projection is

$$\text{proj}_{\vec{u}}(\vec{v}) = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\right) \vec{u}$$

So

$$\text{proj}_{\vec{u}}(\text{proj}_{\vec{u}}(\vec{v})) = \text{proj}_{\vec{u}}\left(\left(\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\right) \vec{u}\right)$$

$$= \frac{\left(\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u}\right) \cdot \overline{u}}{\vec{u} \cdot \vec{u}} \vec{u}$$

$$= \frac{\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} (\vec{u} \cdot \vec{u})}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u} = \text{proj}_{\vec{u}}(\vec{v})$$

17 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 DON'T MARK

Consider the planes in $\mathbb{R}^3$ defined by the equations

$$2x + 2y + 6z = 14 \qquad \text{and} \qquad -3x - 2y - 7z = -16.$$

Find the *vector form* equation of the line of intersection between these two planes.

$$2x + 2y + 6z = 14$$
$$-3x - 2y - 7z = -16$$
$$\downarrow$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 6 & 14 \\ -3 & -2 & -7 & -16 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 7 \\ -3 & -2 & -7 & -16 \end{array}\right] \xrightarrow{3R_1+R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 7 \\ 0 & 1 & 2 & 5 \end{array}\right] \xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5 \end{array}\right]$$

$$x + z = 2$$
$$y + 2z = 5$$

z is a free variable

$$z = t$$

$$x = 2 - t$$
$$y = 5 - 2t$$
$$z = t$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

18 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 **DON'T MARK**

Determine if the matrix A is invertible, and if so find A^{-1}

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 3 & 0 & -1 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 3 & 0 & -1 & 0 & 0 & 1 \end{array}\right] R_1 \Leftrightarrow R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & -1 & 0 & 1 & 0 & 0 \\ 3 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ -3R_1+R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & -3 & -4 & 0 & -3 & 1 \end{array}\right] -R_2+R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right] \begin{array}{l} -\frac{1}{3}R_2 \\ -\frac{1}{2}R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right] \begin{array}{l} -R_3+R_1 \\ -\frac{2}{3}R_3+R_2 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 1 & 0 & -2/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right] -R_2+R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 1 & 0 & -2/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$$\underline{A^{-1} = \begin{bmatrix} 1/6 & 1/6 & 1/6 \\ -2/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & -1/2 \end{bmatrix}} = \frac{1}{6}\begin{bmatrix} 1 & 1 & 1 \\ -4 & 2 & 2 \\ 3 & 3 & -3 \end{bmatrix}$$