Lecture 25

Tuesday, December 1, 2020 3:35 PM

V subspace of $\mathbb{R}^n$

$\mathcal{B} = \{v_1, \dots, v_k\}$ $\underbrace{\text{orthogonal basis}}$

- basis
- $\underbrace{v_i \perp v_j}$ $(i \neq j)$

$$v_i \cdot v_j = 0$$

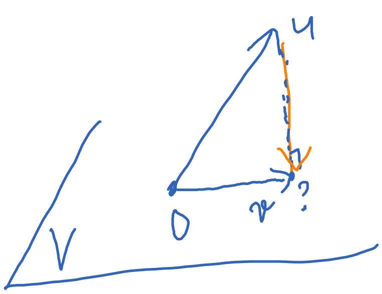
$V = \mathbb{R}^3$

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \leftarrow \text{orthogonal, not orthonormal}$$

$$\uparrow \quad \uparrow \quad \uparrow$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

- Easier to find projection:



$$\left(v = c_1 v_1 + c_2 v_2 + \dots + c_k v_k \right) \cdot v_2$$

$$\underbrace{v \cdot v_2} = c_1 \underbrace{v_1 \cdot v_2}_{0} + c_2 v_2 \cdot v_2 + \dots + c_k \underbrace{v_k \cdot v_2}_{0}$$

$$= c_2 v_2 \cdot v_2$$

$$\left(v = (v - u) + u \right) \cdot v_2$$

$$v \cdot v_2 = \underbrace{(v - u) \cdot v_2}_{0} + u \cdot v_2 = u \cdot v_2$$

$$\Downarrow$$

$$c_1 = \frac{u \cdot v_1}{v_1 \cdot v_1} \qquad c_2 = \frac{u \cdot v_2}{v_2 \cdot v_2}$$

Ex:

$$V = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$u = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

What is $\text{proj}_V u$?

- Easier to find coordinate:

$$\mathcal{B} = \{v_1, \dots, v_k\}$$

u

$$[u]_{\mathcal{B}} =$$

$$\mathcal{B} = \left\{ \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{v_2}, \underbrace{\begin{bmatrix} 3 \\ 6 \\ -5 \end{bmatrix}}_{v_3} \right\} \text{ of } \mathbb{R}^3$$

$$u = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad [u]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

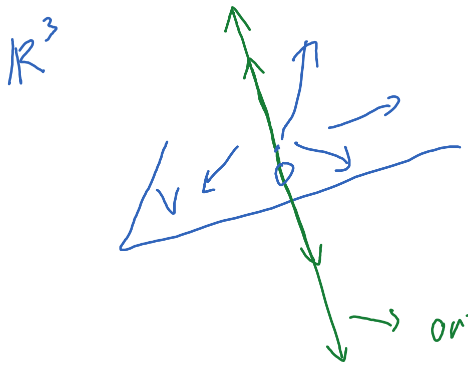
$$(c_1 v_1 + c_2 v_2 + c_3 v_3 = u) \cdot v_1$$

$$c_1 v_1 \cdot v_1 = u \cdot v_1$$

$$\to c_1 = \frac{u \cdot v_1}{v_1 \cdot v_1}$$

$$\begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} | \\ u \\ | \end{bmatrix}$$

Orthogonal complement of a subspace

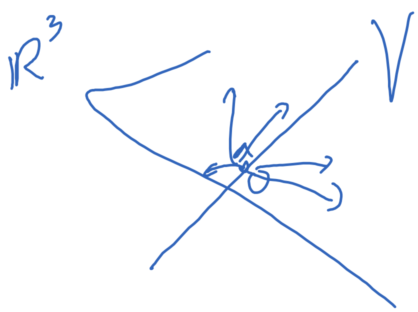


We say that $u \perp V$ if $u \perp v$ for every $v \in V$.

$\longrightarrow$ orthogonal complement of V.

Subspace V of $\mathbb{R}^n$.

$$V^\perp = \{ w \in \mathbb{R}^n : w \perp V \} \quad \text{orthogonal complement of } V$$



Given $V = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$ subspace of $\mathbb{R}^3$

what is $V^\perp$?

$$V^\perp = \{ w \in \mathbb{R}^3 : w \perp V \}$$

$$w_1 \perp V, \; w_2 \perp V \longrightarrow w_1 + w_2 \perp V$$

$w \in V^{\perp}$

$V = \text{span}\{v_1, v_2\}$

$$\left.\begin{aligned} v_1 \cdot w &= 0 \\ v_2 \cdot w &= 0 \end{aligned}\right\} \longrightarrow \underbrace{\begin{bmatrix} - & v_1 & - \\ - & v_2 & - \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} | \\ w \\ | \end{bmatrix}} = \begin{bmatrix} | \\ 0 \\ | \end{bmatrix}$$

$$V^{\perp} = \text{null}(A) \leftarrow$$

$$V = \text{row}(A)$$

Ex.

$$V = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \right\}$$

$$\quad\; v_1 \qquad\quad v_2$$

What vector should we add to V to get an orthogonal basis of $\mathbb{R}^3$?

$$\begin{aligned} v_1 \cdot w &= 0 \\ v_2 \cdot w &= 0 \end{aligned} \qquad \begin{bmatrix} - & v_1 & - \\ - & v_2 & - \end{bmatrix} \begin{bmatrix} | \\ w \\ | \end{bmatrix} = \begin{bmatrix} | \\ 0 \\ | \end{bmatrix}$$