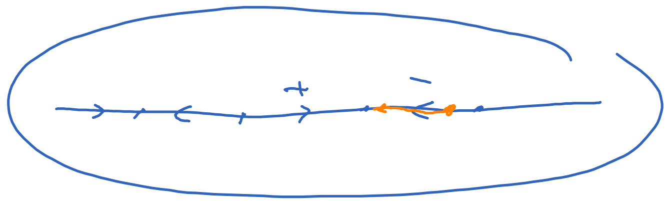
Lecture 27

Tuesday, December 8, 2020 11:44 AM

$y' = f(y)$



$x' = \underbrace{P x}_{f(x)}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

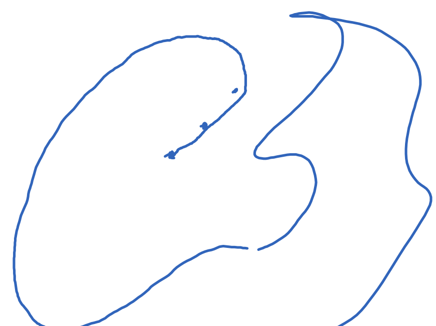
$f(x) = \begin{bmatrix} x_1^2 + x_2 \\ x_2^2 + x_1 \end{bmatrix}$

$\underbrace{y' = f(t, y)}_{\text{nonautonomous}}$

$\underbrace{y' = f(y)}_{\text{autonomous}}$

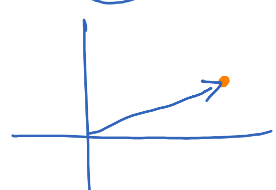
$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x' = P \underbrace{x}_{\text{point on } \mathbb{R}^2}$

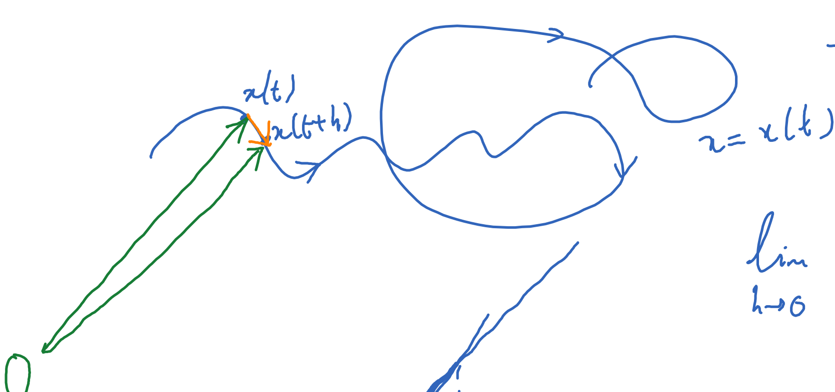


$x = x(t)$

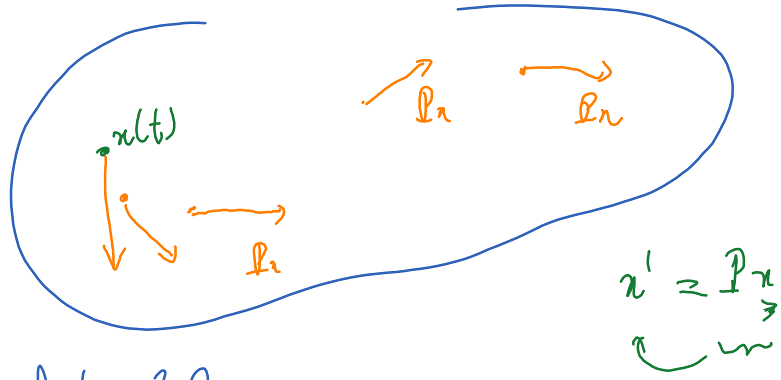
Px ... vector



$\underset{\sim}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$



$$\lim_{h \to 0} \frac{x(t+h) - x(t)}{h} = x'(t) = P x$$



① $P = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

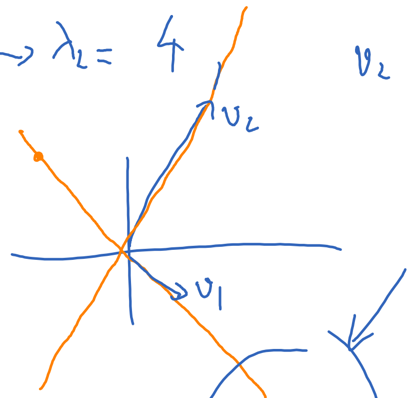
$x^{(1)} = e^{-t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \rightarrow \lambda_1 = -1, \quad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$x^{(2)} = e^{4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix} \rightarrow \lambda_2 = 4 \quad v_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

a) Plot direction field, phase portrait on Mathematica

b) See what is going on on the direction of eigenvectors.

c) Observe the trend of the trajectory when you start at a point outside of the direction of eigenvectors.



$$x = c_1 x^{(1)} + c_2 x^{(2)} = \underbrace{c_1 e^{-t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}} + \underbrace{c_2 e^{4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix}}$$

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② $P_2 = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ $\quad \lambda_1 = 1 \quad v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = 2 \quad v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

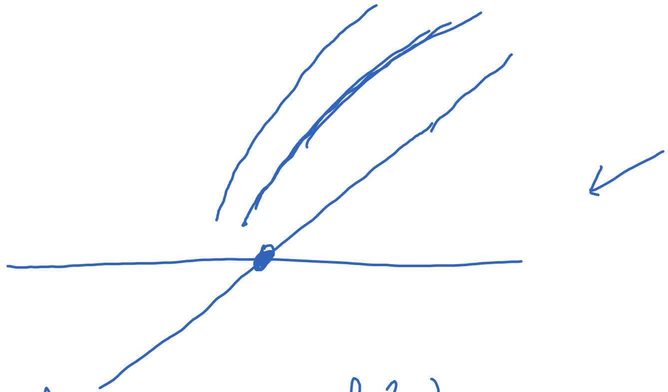
③ $P_3 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ $\quad \lambda_1 = 1+i \quad v_1 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$\lambda_2 = 1-i \quad v_2 = \begin{bmatrix} 1 \\ i \end{bmatrix}$

④ $P_4 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad \lambda_1 = i \quad v_1 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$\lambda_2 = -i \quad v_2 = \begin{bmatrix} 1 \\ i \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$



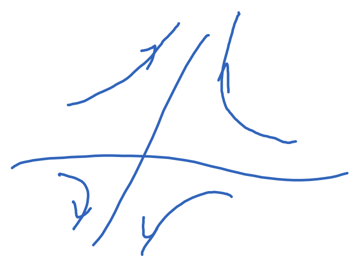
$x' = Ax$

$x(t) = 0$

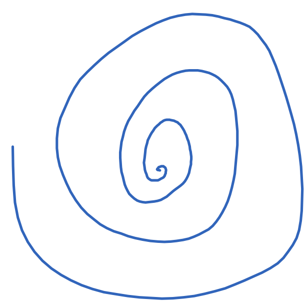
$$x = c_1 e^{t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

λ_1, λ_2 same sign : 0 is called a node.

λ_1, λ_2 diff sign : " " saddle point



λ_1, λ_2 complex, real part $\neq 0$ $\rightarrow$ 0 is spiral point



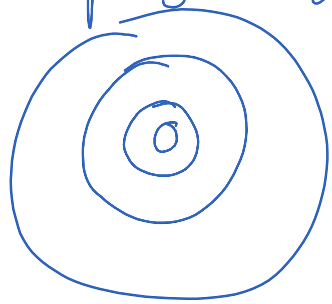
$x' = Ax$

$x' = A(x - \vec{x}_0)$

$x(t) = x_0$

$\lambda_1, \lambda_2 = 1 \pm i$

λ_1, λ_2 are purely imaginary $\quad \lambda_1, \lambda_2 = \pm c i$



0 is center point.