

Access Mathematica and visualize direction fields

1 Get access

If you have not had access to Mathematica yet, you can download Mathematica from this website: <https://home.byu.edu/webapp/softwareDistribution/shop/searchResults/vendor/Wolfram%20Research.htm>

You will be asked to log in with your BYU account. Click on the “Download Now” button on the right. During your installation of Mathematica, you will be asked to create a Wolfram Alpha account. After installing Mathematica on your computer, check your email (the email you used to create your Wolfram Alpha account). You should receive an activation code from Wolfram Customer Support.

2 First experiments

- (1) Type $35/6$, then Shift+Enter.
- (2) Type $N[35/6]$ (note: square brackets), then Shift+Enter.
- (3) Type $Sqrt[2]$ (note: capitalized S). Shift+Enter.
- (4) Type $N[%]$. Shift+Enter.
- (5) Type $Sin[Pi]$ (note: capitalize S and P). Shift Enter.
- (6) Type 34^100 ; (note: semicolon). Shift Enter.
- (7) Type 34^100 (note: without semicolon). Shift Enter.

At this point, you may have noticed that the function N is to evaluate an expression in decimal-point form. Each function’s name is capitalized and used with square brackets (not parenthesis as we normally do on paper). The semicolon is to hold the output. One uses it when output is too long or not of interest. Next, try the following:

- (8) $Exp[1]$. Shift+Enter.
- (9) $Log[2]$. Shift+Enter.
- (10) $f[x_] := Sin[x]+Cos[x]$ (note: the dash after x). Shift+Enter.
- (11) $f[Pi]+f[Pi/4]$. Shift+Enter.
- (12) $Clear[f]$. Shift+Enter.
- (13) $f[Pi]+f[Pi/4]$. Shift+Enter.

The natural logarithm function is named Log in Mathematica (not ln). Exp is the exponential function. The third command is to define a function. The dash is required in order to tell Mathematica that we are defining the function f . The function $Clear$ is to remove a defined variable from the memory. Next, try the following:

- (14) $Plot[Sin[x], \{x,0,2*Pi\}]$. Shift+Enter.
- (15) $Plot[Sin[x], \{x,0,2*Pi\}, Filling\rightarrow Axis]$. Shift+Enter. Note: the arrow is typed as \rightarrow .

(16) `f[x_] := Sin[x]+Cos[x]`. Shift+Enter.

(17) `Plot[f[x], {x,0,2*Pi}, Filling→Axis]`. Shift+Enter. Note: the dash after x is no longer used after the definition of f .

3 Draw direction fields

You can use the command **VectorPlot** to visualize a vector field. A vector field can be thought as a map of arrows: at position (x, y) is placed a vector $(u(x, y), v(x, y))$. The syntax is:

```
VectorPlot[{u[x,y],v[x,y]},{x,a,b},{y,c,d}]
```

For example, the vector field $(u, v) = (x + y, \sin(y))$ is visualized as (Figure 1):

```
VectorPlot[{x + y, Sin[y]},{x, -2, 2}, {y, -2, 2}]
```

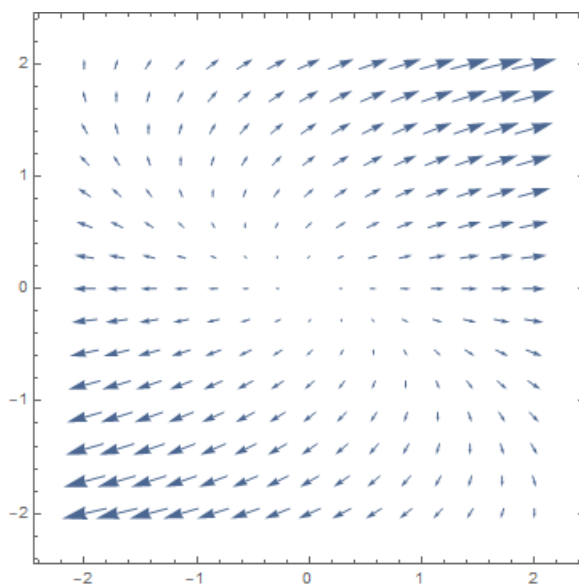


Figure 1

You can see from Figure 1 that the command `VectorPlot`, however, does not show exactly the vector field $(x + y, \sin(y))$. If that were the case then the vector at position $(x, y) = (2, 2)$ would be $(2 + 2, \sin 2) \approx (4, 0.91)$, which is such a long vector. The picture shows an arrow shorter than that! In fact, the command `VectorPlot` shows a scaled version of the vector field: Mathematica automatically scales all arrows by some appropriate factor so that they can be fitted in the box. Although this practice alters the length of arrows, it does not change their directions.

You can also see that arrows in the middle of the box are much shorter than those in the top-right and bottom-left of the box, making it hard to trace curves that start from the middle of the box. A remedy is to scale each arrow by some factor to make it length-one. For example, instead of drawing vector field $(u(x, y), v(x, y))$, we draw the “normalized” vector field

$$\left(\frac{u(x, y)}{\sqrt{u(x, y)^2 + v(x, y)^2}}, \frac{v(x, y)}{\sqrt{u(x, y)^2 + v(x, y)^2}} \right)$$

For example, in the previous example:

```

u[x_, y_] := x + y
v[x_, y_] := Sin[y]
w[x_, y_] := Sqrt[u[x, y]^2 + v[x, y]^2]
VectorPlot[{u[x, y]/w[x, y], v[x, y]/w[x, y]}, {x, -2, 2}, {y, -2, 2}]

```

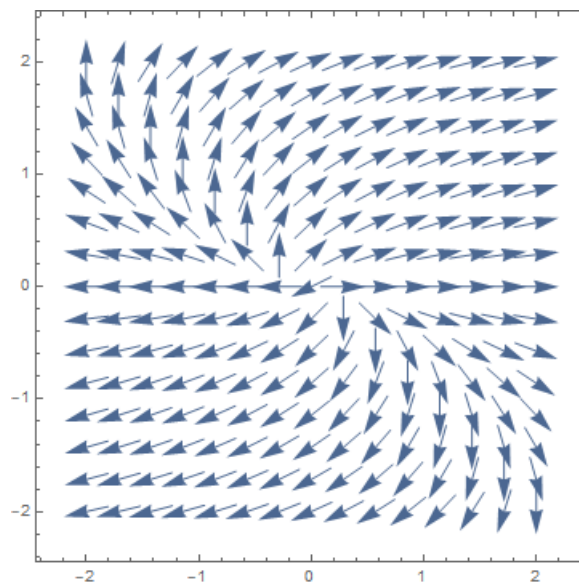


Figure 2

To draw streams directed by the arrows in a vector field, you use the command **StreamPlot**:

```

StreamPlot[{u[x, y]/w[x, y], v[x, y]/w[x, y]}, {x, -2, 2}, {y, -2, 2}]

```

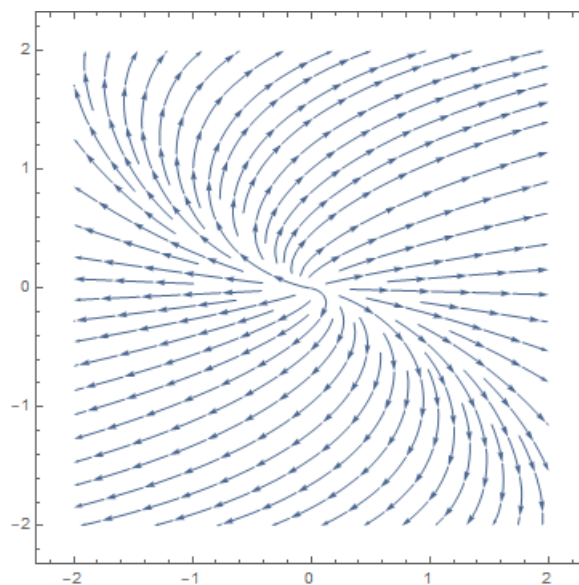


Figure 3

Direction field of an ODE $y' = f(t, y)$ is the vector field $(1, f(t, y))$. Note that the vector $(1, f(t, y))$ has slope equal to $f(t, y)$.

Practice: How do you draw direction field and the integral curves (i.e. the stream lines) of the ODE $y' = -y(3 - ty)$ in the box $(0, 10) \times (-10, 10)$? You might need to normalize the direction field (as explained above) to have a better visual effect.