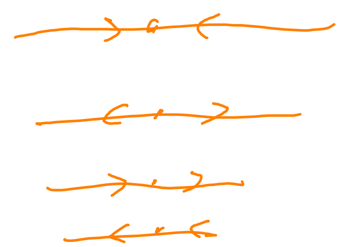
$$Y' = f(Y), \quad Y(t_0) = \boxed{Y_0}$$

Ex:
$$\begin{cases} y_1' = y_1 + y_2^2 \\ y_2' = y_1 y_2 + 2y_2 \end{cases} \qquad f\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_1 + y_2^2 \\ y_1 y_2 + 2y_2 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \vec{0}$$

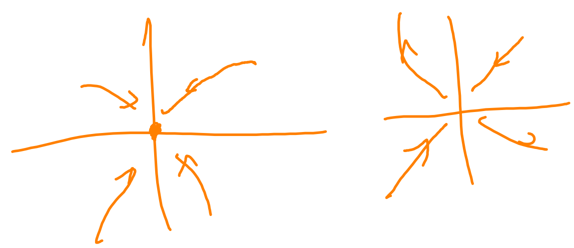
An equilibrium state is the point where $f(Y) = 0$.

Ex:

① $$\begin{cases} y_1' = y_1 + 2y_2 \\ y_2' = 3y_1 + 2y_2 \end{cases}$$



② $$\begin{cases} y_1' = y_1 + y_2 \\ y_2' = 2y_2 \end{cases}$$



③ $$\begin{cases} y_1' = -y_1 \\ y_2' = -y_1 - 2y_2 \end{cases}$$

$$Y' = AY$$
has eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$

This ODE has a fundamental set of sol. $Y^{(1)} = v_1 e^{\lambda_1 t}, \dots, Y^{(n)} = v_n e^{\lambda_n t}$.

$\vec{0}$ is a hyperbolic equilibrium:

If $\lambda_1, \dots, \lambda_n < 0$ then $\vec{0}$ is a stable equilibrium.
If $\lambda_1, \dots, \lambda_n > 0$ then $\vec{0}$ is an unstable equilibrium.
$\Big\}$ $\vec{0}$ is a **node.**

If some of λ_k's are positive, some are negatively, then 0 is semistable equilibrium $\longrightarrow$ $\vec{0}$ is a **saddle point**.

If one of the λ_k's is equal to 0 then $\vec{0}$ is called a **non-hyperbolic** equilibrium.

Ex:

$$Y' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} Y \rightsquigarrow \lambda_1 = -1, \quad \lambda_2 = 4$$

$\vec{0}$ is a saddle point.

$$Y' = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} Y \rightsquigarrow \lambda_1 = 1, \quad \lambda_2 = 2$$

$\vec{0}$ is an unstable node.

$$Y' = \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} \rightsquigarrow \lambda_1 = -1, \quad \lambda_2 = -2$$

$\vec{0}$ is a stable node.