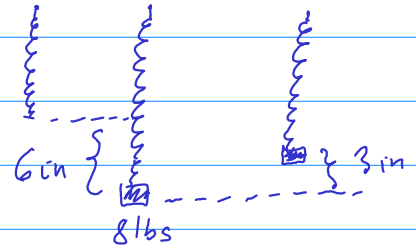
Finish an example in class!



Choose the length unit to be ft, the force unit to be lbs. Then

$k = 16$

$m = 1/4$

Get the equation

$$\frac{1}{4}y'' + 2y' + 16y = 8\sin 8t. \quad (*)$$

$\gamma = 2$ (lbs·s/ft)

$F = 8\sin 8t$

Initial conditions: $y(0) = -1/4$, $y'(0) = 0$.

One can solve (*) by either the undetermined coefficients method or the variation of parameter method.

$$y = y_c + y_p$$

$$y_c = e^{-4t}(c_1\cos 4\sqrt{3}t + c_2\sin 4\sqrt{3}t)$$

Guess $y_p = A\cos 8t + B\sin 8t$.

Plug y_p into (*) to get A and B: $A = -1/2$, $B = 0$.

Thus, $$y = -\frac{1}{2}\cos 8t + e^{-4t}(c_1\cos 4\sqrt{3}t + c_2\sin 4\sqrt{3}t)$$

Using the initial conditions, we get $c_1 = \frac{1}{4}$, $c_2 = \frac{\sqrt{3}}{12}$.

$$y = \underbrace{-\frac{1}{2}\cos 8t}_{\text{perpetuates}} + \underbrace{e^{-4t}\left(\frac{1}{4}\cos 4\sqrt{3}t + \frac{\sqrt{3}}{12}\sin 4\sqrt{3}t\right)}_{\text{dies out over time}}$$

perpetuates: steady state sol. / forced response

dies out over time: transient sol. to the initial value problem