Finish an example in class today (11/01/2021):

$$y''' + 5y'' + 12y' + 8y = e^{-t}, \quad y(0) = y'(0) = y''(0) = 0.$$

We got $$Y = \frac{1}{(s+1)^2(s^2+4s+8)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{Cs+D}{s^2+4s+8}$$

Get B by multiplying both sides by $(s+1)^2$ and then plug $s=-1$: $B = 1/5$

Find A, B, C: plug in 3 values of s to get 3 eqs to solve for A, C, D

On Mathematica: use the command Apart. $\rightsquigarrow$ $A = -2/25$, $C = 2/25$, $D = 1/25$

$$Y = \mathcal{L}\{y\} = -\frac{2}{25}\frac{1}{s+1} + \frac{1}{5}\frac{1}{(s+1)^2} + \frac{1}{25}\frac{2s+1}{s^2+4s+8}$$

$$= -\frac{2}{25}\mathcal{L}\{e^{-t}\} + \frac{1}{5}\mathcal{L}\{te^{-t}\} + \frac{1}{25}\underbrace{\frac{2s+1}{(s+2)^2+2^2}}_{(*)}$$

Note:

$$(*) = \frac{2(s+2)-3}{(s+2)^2+2^2} = 2\underbrace{\frac{s+2}{(s+2)^2+2^2}}_{\mathcal{L}\{e^{-2t}\cos 2t\}} - 3\underbrace{\frac{1}{(s+2)^2+2^2}}_{\frac{1}{2}\mathcal{L}\{e^{-2t}\sin 2t\}}$$

Therefore, $$\mathcal{L}\{y\} = \mathcal{L}\left\{-\frac{2}{25}e^{-t} + \frac{1}{5}te^{-t} + \frac{2}{25}e^{-2t}\cos 2t - \frac{3}{50}e^{-2t}\sin 2t\right\}.$$

$$y = -\frac{2}{25}e^{-t} + \frac{1}{5}te^{-t} + \frac{2}{25}e^{-2t}\cos 2t - \frac{3}{50}e^{-2t}\sin 2t$$