Finish an example given in class (11/05/2021): we need to find the inverse Laplace transform of function

$$Y = \underbrace{\frac{s}{(s^2+1)(s^2+2s+2)}}_{F} + e^{-\pi s}\underbrace{\frac{s}{(s^2+1)(s^2+2s+2)}}_{F} = F + e^{-\pi s}F$$

Partial fractional decomposition:

$$F(s) = \frac{s}{(s^2+1)(s^2+2s+2)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+2} \qquad \left(A = \frac{1}{5},\ B = \frac{2}{5},\ C = -\frac{1}{5},\ D = -\frac{4}{5}\right)$$

$$\mathcal{L}^{-1}\{F\} = A\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + B\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + C\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\} + (D-C)\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$

$$= \frac{1}{5}\cos t + \frac{2}{5}\sin t - \frac{1}{5}e^{-t}\cos t - \frac{3}{5}e^{-t}\sin t = f(t)$$

$$\mathcal{L}^{-1}\{Y\} = \underbrace{\mathcal{L}^{-1}\{F\}}_{f(t)} + \underbrace{\mathcal{L}^{-1}\{e^{-\pi s}F\}}_{(*)}$$

Note that

$$e^{-\pi s}F = e^{-\pi s}\mathcal{L}\{f\} = \mathcal{L}\{u_\pi(t)f(t-\pi)\}$$

Thus,

$$(*) = \mathcal{L}^{-1}\{e^{-\pi s}F\} = u_\pi(t)\, f(t-\pi).$$

Therefore,

$$y = \mathcal{L}^{-1}\{Y\} = f(t) + u_\pi(t) f(t-\pi)$$

$$y = \frac{1}{5}\cos t + \frac{2}{5}\sin t - \frac{1}{5}e^{-t}\cos t - \frac{3}{5}e^{-t}\sin t$$

$$+ u_\pi(t)\left(-\frac{1}{5}\cos t - \frac{2}{5}\sin t + \frac{1}{5}e^{-(t-\pi)}\cos t + \frac{3}{5}e^{-(t-\pi)}\sin t\right)$$